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TESTING PROPOSAL

FOR

SINE WAVE

TESTING EQUIPMENT

997004

STATINTL

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I Theoretical Background

I-1 The Contrast Transfer Function

For the purpose of introducing the nomenclature, we will first set down some of the well known relations in image formation.

The intensity distribution in the object plane is given by:

$$o(x,y) \tag{I-1}$$

The spatial frequency content of the object $O(\nu_1, \nu_2)$ is then given by the Fourier transform:

$$O(\nu_1, \nu_2) = \iint_{-\infty}^{+\infty} o(x,y)\, e^{-2\pi i (x\nu_1 + y\nu_2)}\, dx\, dy \tag{I-2}$$

When we consider only one object point in the object plane we can define the amplitude in the wavefront after passing the optical system as $A(\nu_1, \nu_2)$ (in a given reference plane). The amplitude in the diffraction pattern (in the chosen image plane) is $a(x,y)$. The image spread function of this given object point is denoted by $t(x,y)$. We then have the following relation:

$$t(x,y) = \left| a(x,y) \right|^2 \tag{I-3}$$

and the contrast transfer function $T(\nu_1, \nu_2)$ is then

$$T(\nu_1, \nu_2) = \frac{\iint_{\infty} A(\mu_1, \mu_2)\, A^*(\mu_1 + \nu_1, \mu_2 + \nu_2)\, d\nu_1\, d\nu_2}{\iint_{\bigcirc} \left| A(\nu_1, \nu_2) \right|^2\, d\nu_1\, d\nu_2} \tag{I-4}$$

We now define the image intensity distribution by

$$i(x,y) \qquad \text{(I-5)}$$

Then the Fourier transfer $I(\nu_1,\nu_2)$ of the image is

$$I(\nu_1,\nu_2) = \iint_{-\infty}^{+\infty} i(x,y) e^{-2\pi i (x\nu_1 + y\nu_2)} dxdy \qquad \text{(I-6)}$$

We then have the basic relationship:

$$I(\nu_1,\nu_2) = T(\nu_1,\nu_2) \quad O(\nu_1,\nu_2) \qquad \text{(I-7)}$$

The purpose of our proposal is to build a simple instrument with which to measure $T(\nu_1,\nu_2)$.

I-2 Sinusoidal Targets Versus Square Wave Targets

There are various methods of measuring $T(\nu_1,\nu_2)$. Many of these are based on using sinusoidal targets. Assuming that these targets are infinitely long in one direction, our problem reduces to a one dimensional problem. Equation I-1 reduces to

$$o(x) = b_{o,i} + b_i \cos 2\pi \nu_i x_i \qquad \text{(I-8)}$$

The Fourier transform $o(\nu_i)$ of this object consists of only two terms: the d.c. component b_{o_i} and the amplitude of the fundamental b_i.

From the definition of $T(\nu)$ (I-4), we see that $T(o)$ always equals one. The Fourier transform of the image is therefore

$$I(\nu_i) = b_{o,i}\delta(\nu) + \frac{1}{2} T(\nu_i) b_i \left[\delta(\nu + \nu_i) + \delta(\nu - \nu_i)\right] \qquad \text{(I-9)}$$

and the image intensity

$$i(x') = b_{o_i} + \frac{1}{2} T(\nu_i) b_i \left(e^{2\pi i \nu_i' x'} e^{-2\pi i \nu_i' x'}\right) \qquad \text{(I-10)}$$

In general $T(\nu_i)$ is complex. In writing

$$T(\nu_i) = \tau_i e^{i2\pi\varphi_i} \qquad \text{(I-11)}$$

we find for our image

$$i(x') = b_{oi} + \tau_i b_i \cos 2\pi(\nu_i' x_i' + \varphi_i)$$

We will now define dc modulation V of an object:

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \qquad \text{(I-12)}$$

For our sinusoidal object we have: $I_{max} = b_{o,i} + b_i$ and $I_{min} = b_{o,i} - b_i$ and therefore:

$$V_{obj} = \frac{b_i}{b_{oi}} \qquad \text{(I-13)}$$

For the image of this object we find

$$V_{image} = \tau_i \frac{b_i}{b_{oi}} = \tau_i V_{obj} \qquad \text{(I-14)}$$

It is this relation that opens the way to an easy measurement of τ_i as will be shown further on. We will also come back to the phase angle φ.

If we now replace the sinusoidal target with a bar target we have (see Fig. I-1)

$$o(\nu) = \frac{b_o}{2} + \frac{4b_1}{\pi}\left[\cos 2\pi\nu x - \frac{1}{3}\cos 6\pi\nu x + \frac{1}{5}\cos 10\pi\nu - \cdots\right] \qquad \text{(I-15)}$$

with $\nu = \frac{1}{p}$

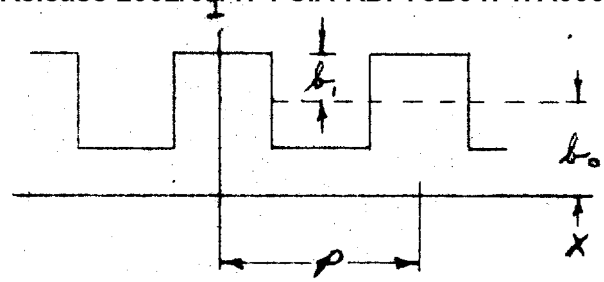


Fig. I-1

Let us now assume that we use a value for p_1 such that the frequency $3\,\nu_1$ falls outside the resolving power limit of the lens. We then have for the image

$$i(x') = b_0 + \tau_{p_1} \frac{4b_1}{\pi} \cos 2\pi(\nu_{p_1} x + \varphi_{p_1}) \qquad \text{(I-16)}$$

The modulation of the object is

$$V_{obj} = \frac{(b_0 + b_1) - (b_0 - b_1)}{(b_0 + b_1) + (b_0 - b_1)} = b_1/b_0$$

For the modulation of the image we find:

$$V_{image} = \frac{\left(b_0 + \tau_{p_1} \frac{4b_1}{\pi}\right) - \left(b_0 - \tau_{p_1} \frac{4b_1}{\pi}\right)}{\left(b_0 + \tau_{p_1} \frac{4b_1}{\pi}\right) + \left(b_0 - \tau_{p_1} \frac{4b_1}{\pi}\right)} = \frac{4}{\pi} \tau_{p_1} \frac{b_1}{b_0} \qquad \text{(I-17)}$$

or

$$V_{image} = \frac{4}{\pi} \tau_{p_1} \; V_{obj}$$

When there is no phase shift i.e. all $\varphi_p = 0$, the measurement of all frequencies are simple. Our image now is:

$$\iota(x') = b_0 + \tau_{p_1} \frac{4b_1}{\pi} \cos 2\pi \nu_{p_1} x - \tau_{p/3} \frac{4b_1}{3\pi} \cos 2\pi 3 \nu_1 x + ---$$

The modulation of this image is

$$V_{image} = \frac{\left\{b_0 + \frac{4b_1}{\pi}\left[\tau_{p_1} - \frac{\tau_{p_1/3}}{3} + ---\right]\right\} - \left\{b_0 - \frac{4b_1}{\pi}\left[\tau_{p_1} - \frac{\tau_{p/3}}{3} + ---\right]\right\}}{\left\{b_0 + \frac{4b_1}{\pi}\left[\tau_{p_1} - \frac{\tau_{p_1/3}}{3} + ---\right]\right\} + \left\{b_0 - \frac{4b_1}{\pi}\left[\tau_{p_1} - \frac{\tau_{p/3}}{3} + ---\right]\right\}} =$$

$$= \frac{4b_1}{\pi b_0}\left[\tau_{p_1} - \frac{\tau_{p/3}}{3} + ---\right] = \frac{4}{\pi}\left[\tau_{p_1} - \frac{\tau_{p/3}}{3} + ---\right] V_{obj} \qquad (I-18)$$

We can already see that this can run into real problems if we have very fluctuating transfer curves, and formula I-18 can become invalid when $\tau_{p/3}$ is much larger than τ_{p_1}. We will not go further into this problem since the solution is determined by the way we propose to measure and will be dealt with later.

but in general $\tau_{p_1} > \tau_{p/3}$

Before we leave our theoretical considerations we would like to treat first the case of bar targets with a finite number of bars.

Let us assume an odd number of bars, numbered from -N to +N. The total number of bars is then 2N+1. The Fourier transform of the bar numbered o is

$$\frac{\sin \frac{1}{2}\pi \; \nu/\nu_o}{\frac{1}{2}\pi \; \nu/\nu_o} \tag{I-19}$$

where ν_o represents the line frequency of the target. The Fourier transform of the pair of bars (-n,+n) is given by

$$2 \cos 2\pi n \frac{\nu}{\nu_o} \frac{\sin \frac{1}{2}\pi \nu/\nu_o}{\frac{1}{2}\pi \nu/\nu_o} \tag{I-20}$$

An addition now gives for the Fourier transform of the entire target consisting of 2N + 1 bars.

$$\frac{\sin \frac{1}{2}\pi \nu/\nu_o}{\frac{1}{2}\pi \nu/\nu_o} + 2\sum_{1}^{N} \cos 2\pi N \nu/\nu_o \frac{\sin \frac{1}{2}\pi \nu/\nu_o}{\frac{1}{2}\pi \nu/\nu_o} \tag{I-21}$$

We can use this series to investigate how many bars we need as a minimum in order to be sure that our measured results will be the same (within a given tolerance) as the results with an infinitely long bar target ($N=\infty$).

The series may be cut off when the factor $\cos 2\pi N \nu/\nu_o$ fluctuates fast enough to make its contribution inappreciable. The range of interest in ν is given by

$$\Delta\nu = 0.1225\,\nu_o$$

representing a one-target frequency difference in an Air Force standard resolving power chart. We require that the cosine factor goes through one full cycle in this interval

$$2\pi N \frac{\Delta\nu}{\nu_0} = 2\pi$$

or

$$N = \frac{\nu_0}{\Delta\nu} = 8$$

This indicates that a target of at least 15 lines must be used under these circumstances, because we can apparently neglect the term in the series with N=8.

From Equ. I-21 we can also easily derive the Fourier transform of the three bar target by taking N=1. We find

$$\left[1 + 2 \cos 2\pi \nu/\nu_0 \right] \frac{\sin \frac{1}{2}\pi \nu/\nu_0}{\frac{1}{2}\pi \nu/\nu_0} \qquad \text{(I-22)}$$

In appendix II we outline our computer program to compute the intensity in the image of a three-bar target formed with a lens of which the transfer function is given.

II Measurement Techniques

II-1 Targets with Variable Modulation

A way to make a target with variable modulation is shown in Fig. II-1.

Two bundles from a single light source are directed towards a half-silvered mirror. Before reaching this half-silvered mirror the following items are placed in the beam:

a) In each beam a diffusing screen (1) is inserted.

b) In one beam a polarizer (2a) is inserted with its polarizing direction as indicated in the plane of the drawing.

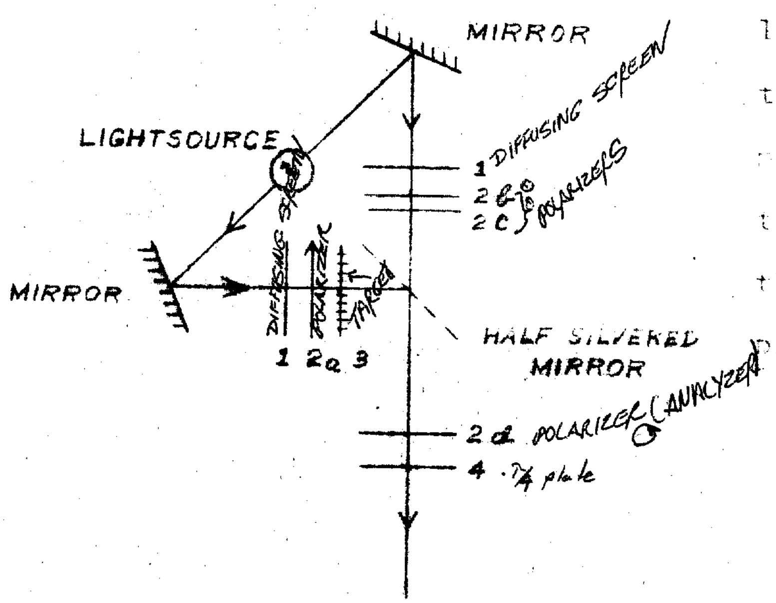


Fig. II-1

In the other beam we insert two polarizers (2b) and (2c), both of which, for the time being are rotated in such a position that the polarizing directions for both is perpendicular to the plane of the drawing. (The reason for two polarizers will become clear later).

c) In the first beam we also place a target (3). For the time being we will not specify the type of target.

d) After the two beams are united we insert a polarizer (2d). This polarizer can be rotated to any desired position. Together with this polarizer we can rotate a quarter-λ plate 4. The relation between the polarizer (2d) and the quarter-λ plate 4 is such that the light emerging from this combination is circularly polarized. We will call the polarizer (2d) from here on the analyzer.

Let us now consider a target with some periodicity in it. With this we mean either a three-bar target, a (2N+1) bar target, or a sinusoidal target with at least a few periods on it.

Let us suppose the maximum intensity in the target is I_a and a minimum of I_c. The maximum intensity in the other bundle is I_b.

Now if for a certain position of the analyzer a fraction λ is transmitted from the light coming from the target, then a fraction $(1-\lambda)$ is transmitted from the other bundle. Therefore coming out of the instrument

$$I_{max} = I_a\lambda + I_b(1-\lambda) = I_b + (I_a-I_b)\lambda$$

$$I_{min} = I_c\lambda + I_b(1-\lambda) = I_b + (I_c-I_b)\lambda \qquad \text{(II-1)}$$

The average is

$$\frac{I_{max} + I_{min}}{2} = \frac{2I_b + \left(I_a - 2I_b + I_c\right)\lambda}{2} \qquad \text{(II-2)}$$

In order to make this average constant we have to adjust the various intensities according to:

$$I_a - 2I_b + I_c = o \quad \text{or} \quad I_b = \frac{I_a + I_c}{2} \qquad \text{(II-3)}$$

The modulation of the target as seen coming through the instrument is

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{(I_a - I_c)\lambda}{2I_b + (I_a - 2I_b + I_c)\lambda}.$$

However, with (II-3) this becomes:

$$V = \frac{(I_a - I_c)\lambda}{2 I_b} = \frac{(I_a - I_c)}{(I_a + I_c)}\lambda \qquad \text{(II-4)}$$

The modulation of the target above is:

$$V_{target} = \frac{I_a - I_c}{I_a + I_c}$$

With this relation we have for II-4:

$$V = \lambda \cdot V_{target} \qquad \text{(II-5)}$$

With this arrangement then the modulation of the target as seen coming through the instrument can therefore be continuously changed between V_{target} and zero.

In order to satisfy relation (II-3) we put in our instrument the polarizer (2_b). By changing the position of polarizer (2_c) we can get any intensity between I_a and zero.

From Equation (II-3) it is clear that I_b is always smaller than I_a and we can, therefore, always fulfill the condition with this setup.

For the analysis of the precisions required see Appendix 1.

II-2 Visual Measuring Techniques

Before we can specify any specific measuring technique we will have to define what the purpose of our measuring is. It is by no means always necessary to measure the complete transfer function, including phase shifts of the equipment alone.

For instance in viewers, using screens to present the image, it seems of the utmost importance that the instrumentation as a whole, including the observer is included in the measuring technique. In other instances, it will be important to get the complete transfer function, with as much data as possible.

We will therefore describe various measuring techniques.

II-2-1 Measurements Which Include the Transfer Function of the Eye

Since the transfer function of the eye will have a great influence on all types of viewers using a screen, and since this function itself is by no means a constant, we will start with a method that will tell the P.I. what he can see, using a particular piece of equipment. In this test he can use all aids that he might use in connection with this equipment, for example loops.

In place of the film to be viewed we put a target as shown in Fig. II-1. This target can have its own light source, or can be illuminated with the light source provided by the viewer. (In the latter case the diffusing screens 1 should be removed). The maximum intensity will be cut by a factor of 1/2 and this may be objectionable. On the other hand, the viewer illumination may not be fully incoherent (viewers with condensors) and it might therefore be preferable to use the viewers illumination system.

With the modulation of the target adjusted to one, the P.I. determines the highest frequency he can resolve. He then takes a target with a lower frequency and adjusts the modulation to such a value that this target is just visible and notes the frequency and the visibility setting. Noting that he has adjusted the modulation of the image on his retina to a minimum (V_{min}) we find from relation I-14

$$\tau_i = \frac{V_{min}}{V_{object}} \qquad \text{(II-6)}$$

We note that we do not have the value of V_{min} (and this is also a function of the frequency of the target). We note also that the value of τ_i completely describes what will be visible in this equipment for this observer.

The value of τ_i will depend on the type of target. The relation between long square-wave targets and sinusoidal targets is easily found mathematically as described above. However, when we use bar targets with a few bars the situation changes drastically. However, for judgement of the actual performance, measurements with three and two bars may be very informative. See, for instance, the intensity distribution of a three-bar target in an aerial image formed by a lens with aberrations as shown in Appendix 2. We therefore suggest including this type of measurement in any equipment to be built.

II-2-2 Objective Measurement of Transfer Function

It is possible to change the setup so as to make the measurement objective. In Fig. II-2, box I represents the object as shown in Fig. II-1 and is located in such a way as to put the target

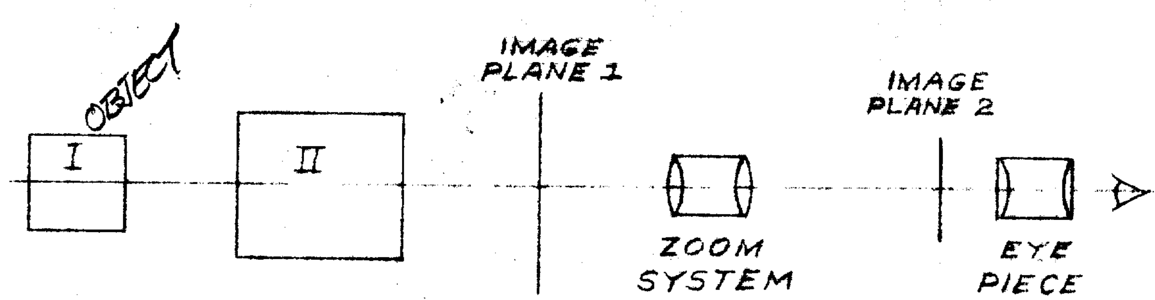


Fig II-2

in the object plane of the optical system (II) under test. The image is formed in the image plane 1, which in turn is imaged in image plane 2 by a zoom system. For each frequency the magnification is adjusted so as to give an image with a fixed frequency in image plane 2.

In principle we now can adjust V_{object} in such a way as to have it just "resolved". Since for all frequencies in the object, the eye is represented with the same frequency, and if we also adjust the illumination of the targets in I such that the illumination level presented to the retina is the same for all frequencies then V_{min} on the retina is a constant. We can now measure the whole transfer function except for a constant V_{min}. The value of V_{min}, however, can be found easily by various methods.

We propose to make the observation slightly differently. This method will eliminate the need for a precise adjustment of the illumination level, will fix the value of V_{min}, will make the measurement applicable to the fundamental frequency only and give a sensitive method of measuring the non-linear phase shifts.

In the image in image plane 2 we will always have an image with a small modulation in the final measurement position. Suppose we have the intensity pattern with vertical lines from the target and for the fundamental frequency the intensity distribution of the image in the horizontal direction is given by

$$I\,(x) = a + b \cos 2\pi \nu_o x \tag{II-7}$$

where ν_o is the fixed frequency in image plane 2. The modulation of this image is

$$V = \frac{b}{a} \tag{II-8}$$

Suppose we now pass the lower half of the field through a filter that has a transmission given by:

$$T = \gamma\,(1 - 2\,\frac{b}{a} \cos 2\pi \nu_o\, x) \tag{II-9}$$

Then the intensity in the half of the image passing through the filter is given by

$$I'(x) = \gamma(a+b \cos \pi \nu_o x)\ (1 - 2\,\frac{b}{a} \cos 2\pi \nu_o\, x)$$

Since b/a is small, say of the order of 0.02 we can write for this

$$I'(x) = \gamma\,(a - b \cos 2\pi \nu_o\, x) \tag{II-10}$$

In this equation γ is a constant which we will give a value

$$\gamma = 1 - 2\,\frac{b}{a} \tag{II-11}$$

If we now cover the other half of image plane 2 with a filter with a transmittance γ, we have in both halves of the field of view of the eyepiece two identical images, however shifted by π. We can now easily optically shift one half of our field over half a period and then have an image that for the fundamental is absolutely identical.

When we fix the value b/a for our filters and pass an image through it with a different modulation, this difference will show up in our eyepiece. The only time we do not see any intensity differences for the fundamental frequency between the top and bottom part of the field, is when the modulation of our image is b/a. In other words, the value V_{min} = b/a is directly built into our equipment and is fixed for all frequencies.

When we use bar targets with N equal to 15, a slight misadjustment of the zoom system will show up as a Moiré pattern in the lower part of the field.

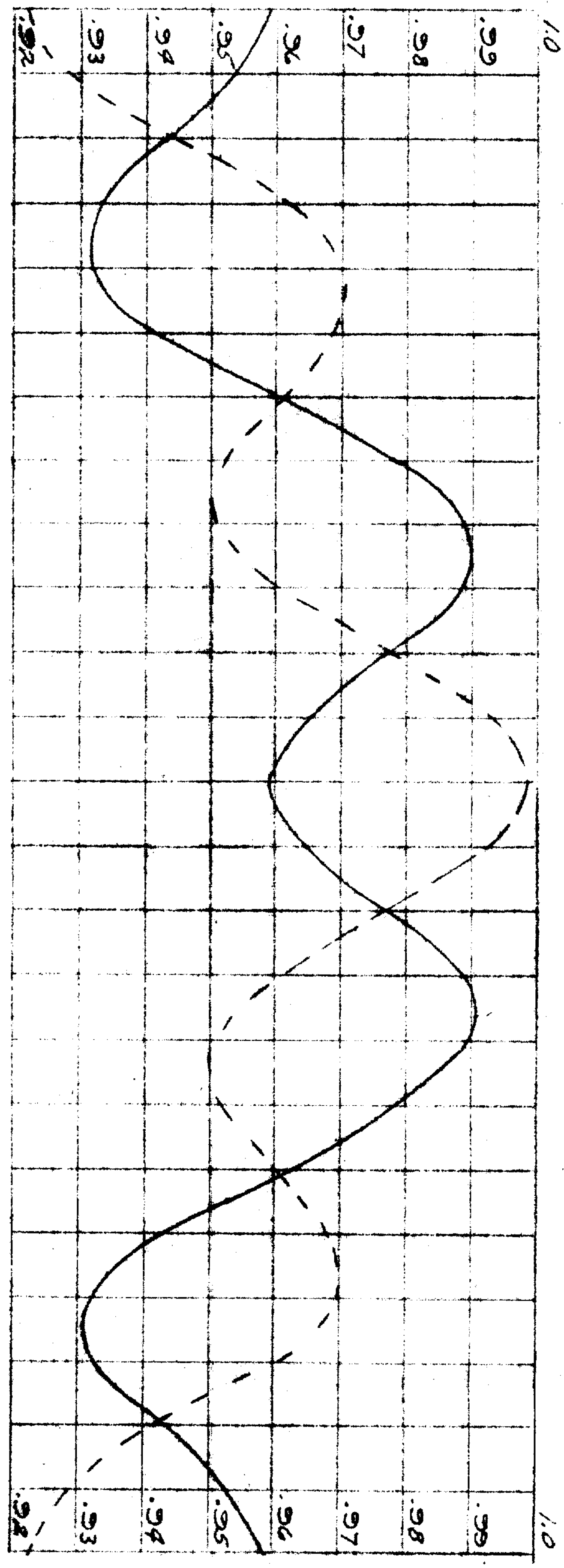
When all our bar targets are provided with a transparent line above the target and our filter in image plane 2 with a line, we can for each target position the filter with the modulation of the target equal to one in the same relative position, in respect to the target. When for a frequency ν a non-linear phase shift occurs, we will have a phase shift between our sinusoidal target and the image which will result in a phase shift between upper and lower target image. By shifting the sinusoidal target to correct for this we find directly the non-linear part of the phase shift. (Linear phase shifts do not affect the image quality of the system under test.)

The higher harmonics that might be present can now easily be recognized and ignored by the eye. There are only higher harmonics in the first third of the frequency range. In normal cases of "well-behaved" transfer functions they will always be small. When large amounts of higher harmonics are present we can use two techniques.

In order to investigate the possibilities here let us take the case where two harmonics are available in the image.

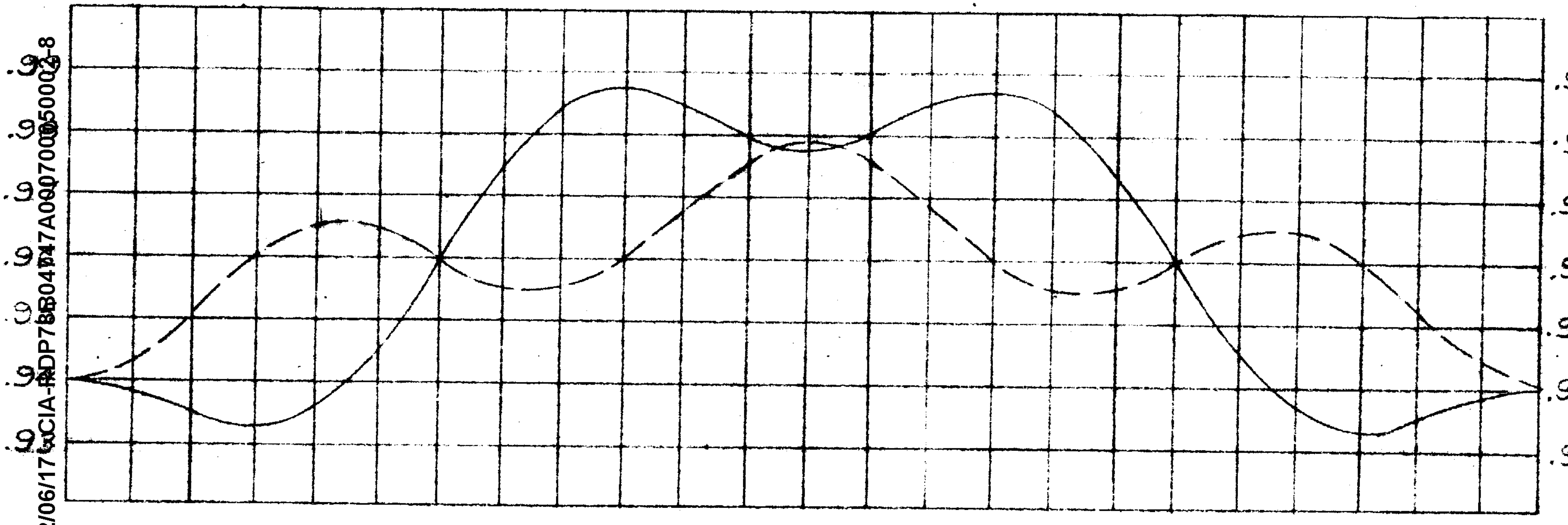
$$I(x) = \bar{a}\left\{1+\bar{b}\cos 2\pi\nu_o x+\frac{\bar{c}}{3}\cos 2\pi(3\nu_o x+\alpha x)\right\} \qquad \text{(II-12)}$$

For the case where $\bar{c} = 3\bar{b}$ (which for a square-wave target is certainly a large increase in the transfer function for the higher frequencies) we have plotted in Fig. II-3 the resultant intensities in both halves of the field when the correct adjustment is made (here $\alpha = o$). In Fig. II-4 and II-5 we have plotted the cases where $\bar{b}/\bar{a}$ are 0.01 and 0.03, or a setting error of 1% is made. It seems perfectly possible to distinguish with the eye within this margin of error.



IN PHASE $\frac{d}{a}$ = .02

FIG II-3



IN PHASE $\frac{b}{a} = .01$

FIG II - 4

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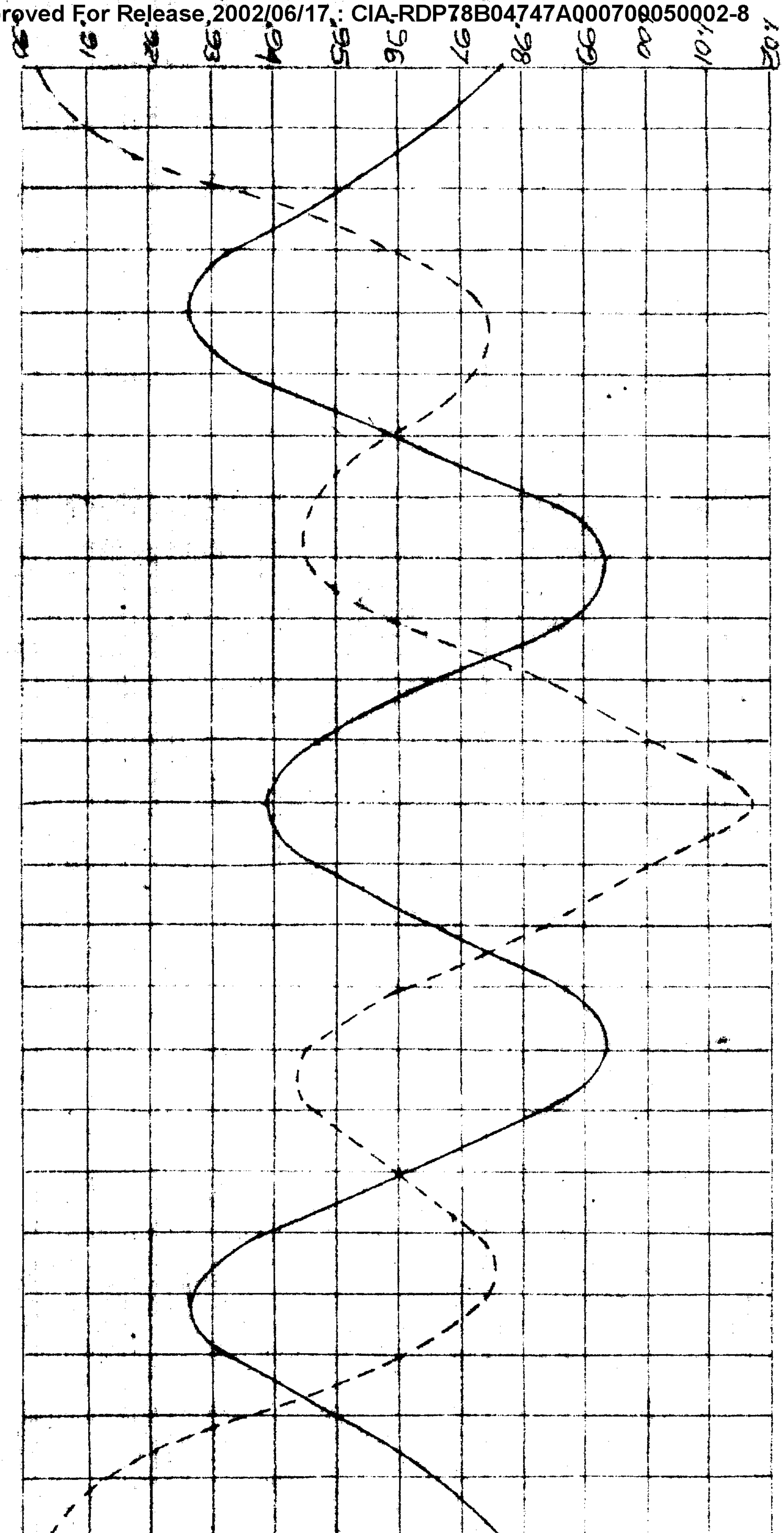


In Phase $\frac{b}{a} = .03$

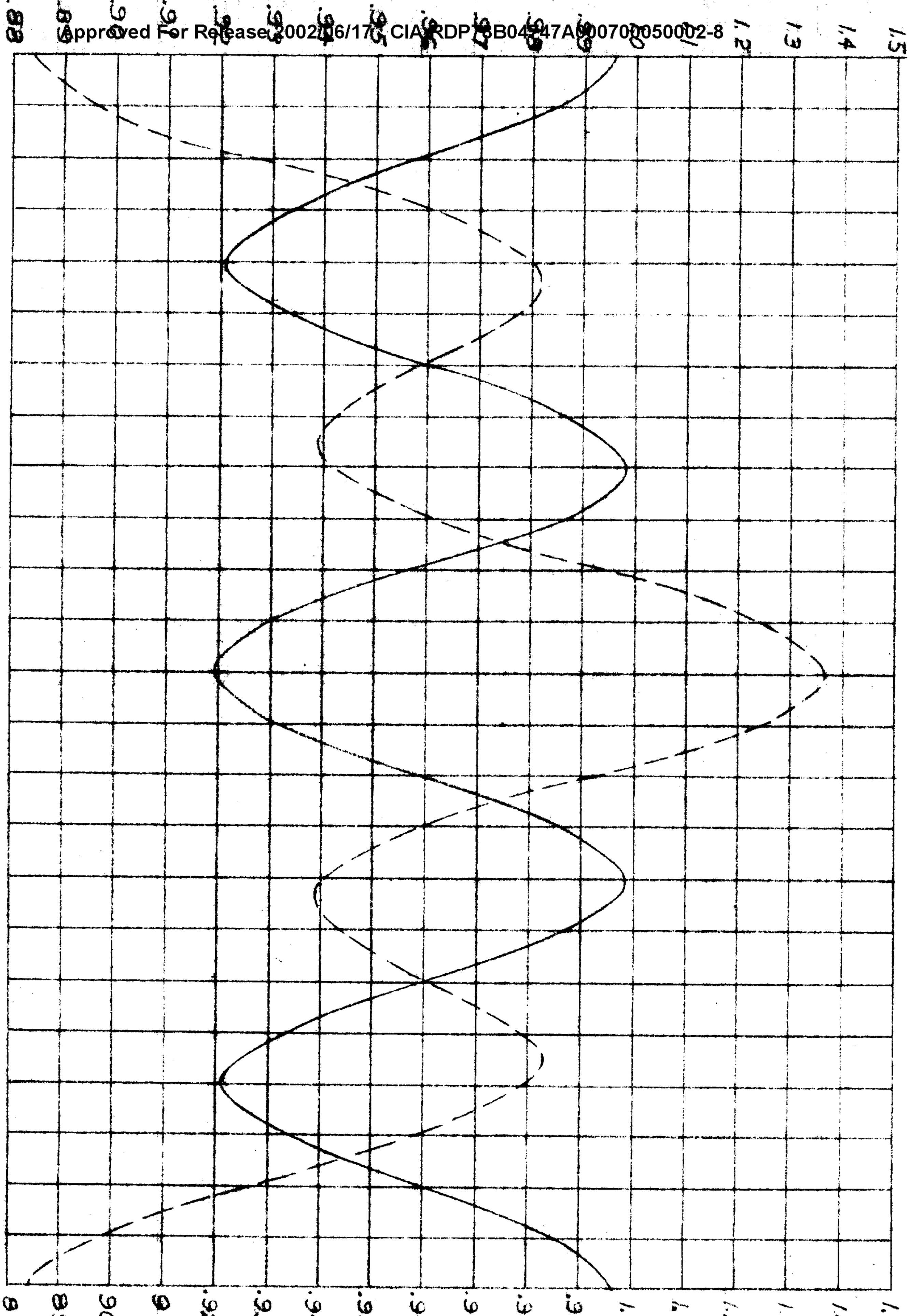
Fig II-5

However this suggests a completely different way of measuring or checking the results acquired in the above methods. When we increase $\bar{b}/\bar{a}$ to 0.04 we will have a complete cancellation of our first harmonic. In this case only the second harmonic is shown in one half of the field. (This second harmonic is lightly modified by the transmission of the filter; however this effect is very small and if desired can be corrected for). The criterion then is that the second harmonic is of equal intensity in all its maxima and minima (see Fig. II-6). When direct judgment by eye is difficult, the phase shift between the upper and lower half can be made adjustable, the sinewave filter inserted in both halves of the field and direct comparison of adjacent intensities will be possible. This method can be carried out with many more harmonics available in the image.

Before leaving this subject it is useful to investigate the precision of the measurement of the first harmonic a little further. Let us assume a signal

$$I(x) = a' \left\{ 1 + \frac{b'}{a'} \cos 2\pi(\nu_0 x + \alpha) \right\} \qquad \text{(II-13)}$$

Let this signal be passed by a filter given by II-9 and II-11. The transmitted intensity distri-



$\frac{\delta}{a} = .04$

FIG. II-6

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bution is then

$$I_1'(x) = I\ (x)\ T(x) \approx \gamma a' \left\{ 1 + \frac{b'}{a'} \cos 2\pi(\nu_0 x + \alpha) - 2\ \frac{b}{a} \cos\ 2\pi \nu_0 x \right\} \qquad \text{(II-14)}$$

This intensity is displayed in one-half of the field of view and compared with the other half of the field of view where the intensity distribution is:

$$I_2'\ (x) = \gamma a' \left\{ 1 - \frac{b'}{a'}\ \cos\ 2\pi(\nu_0 x + \alpha) \right\} \qquad \text{(II-15)}$$

The two signals can be represented by vectors as shown in Fig. II-7

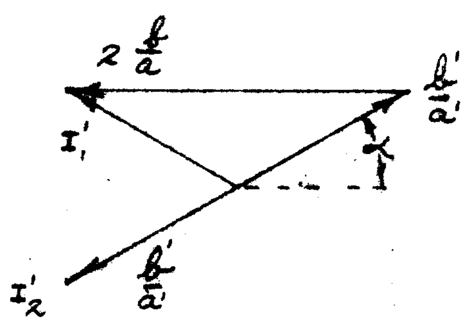
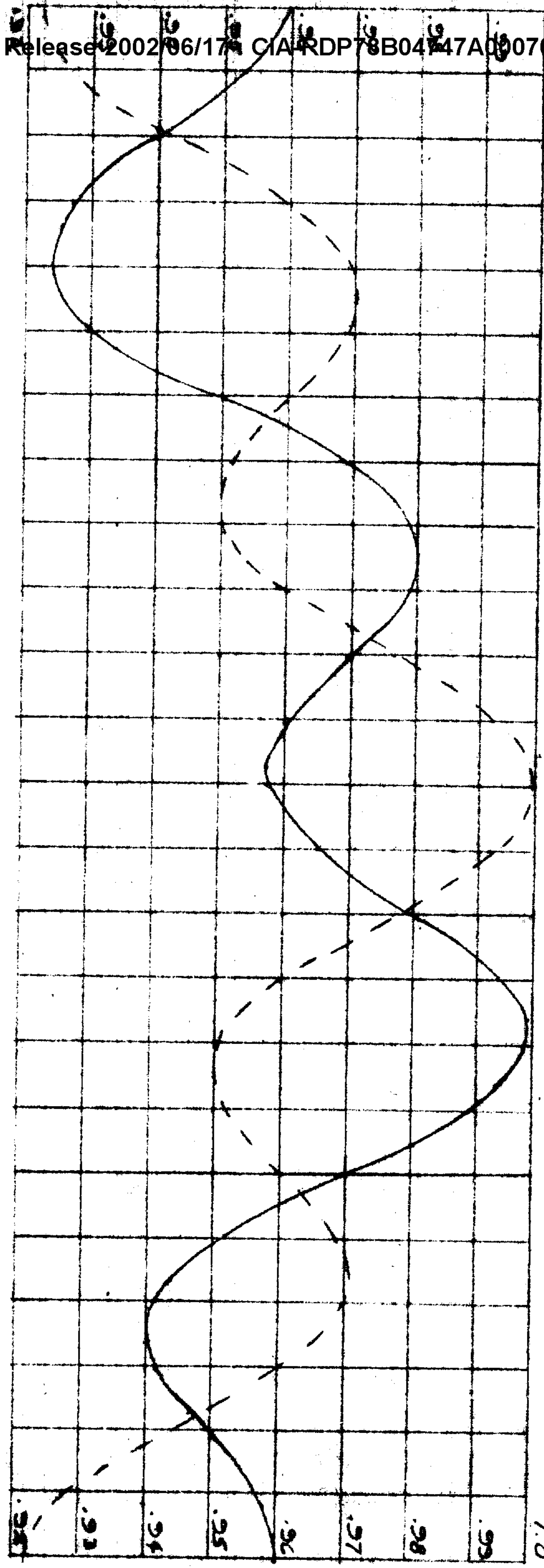


Fig. II-7

The phase difference between the two signals is 2 α. When the phase angle is adjusted the difference in intensity between the two signals is $2\ (\frac{b}{a} - \frac{b'}{a'})$ where the factor of 2 in both phase angle and intensity help the precision of the measurement (The factor of 2 in amplitude is clearly seen in Figs. II-3, 4, 5 and 6. In Fig. II-8 we have introduced a 15° phase angle between the filter and the signal as used in Fig. II-3.



FILTER 15° OUT OF PHASE

$\frac{\delta}{\lambda} = .02$

FIG II-8

III-1 Photographic Tests

After the above descriptions, it is clear that the minimum modulation of the object for each frequency is easily determined for a given film.

It is also clear that by first measuring the transfer curve of the optical parts of the equipment, the transfer curve of the film above can be determined.

At this point we wish to note that transfer functions for films are only usefull when targets with long lines are used. Basically they do not give enough information to determine the capabilities of small details (small in two dimensions) to be resolved. Fundamentally this is a problem involving statistical computations.

IV-1 Experimental Equipment Already Built by [illegible]. L.

To test the above approach a simple model of a tester was built. (See Fig. IV-1 and 2). Since we were primarily interested in the response of optical systems to three-bar targets, this equipment has only three-bar targets. The results of this equipment are shown in Fig. IV-3.

It was also used to test the response of the eye and some known lenses.

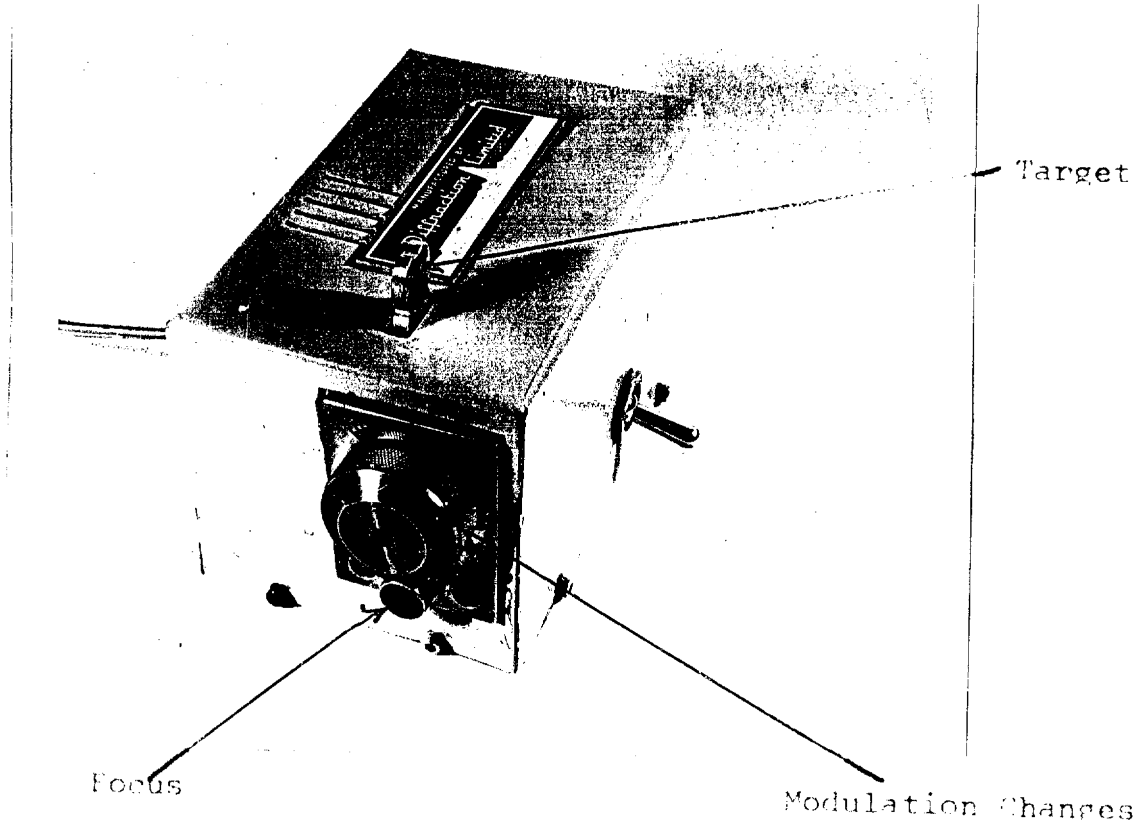


Fig. IV-1

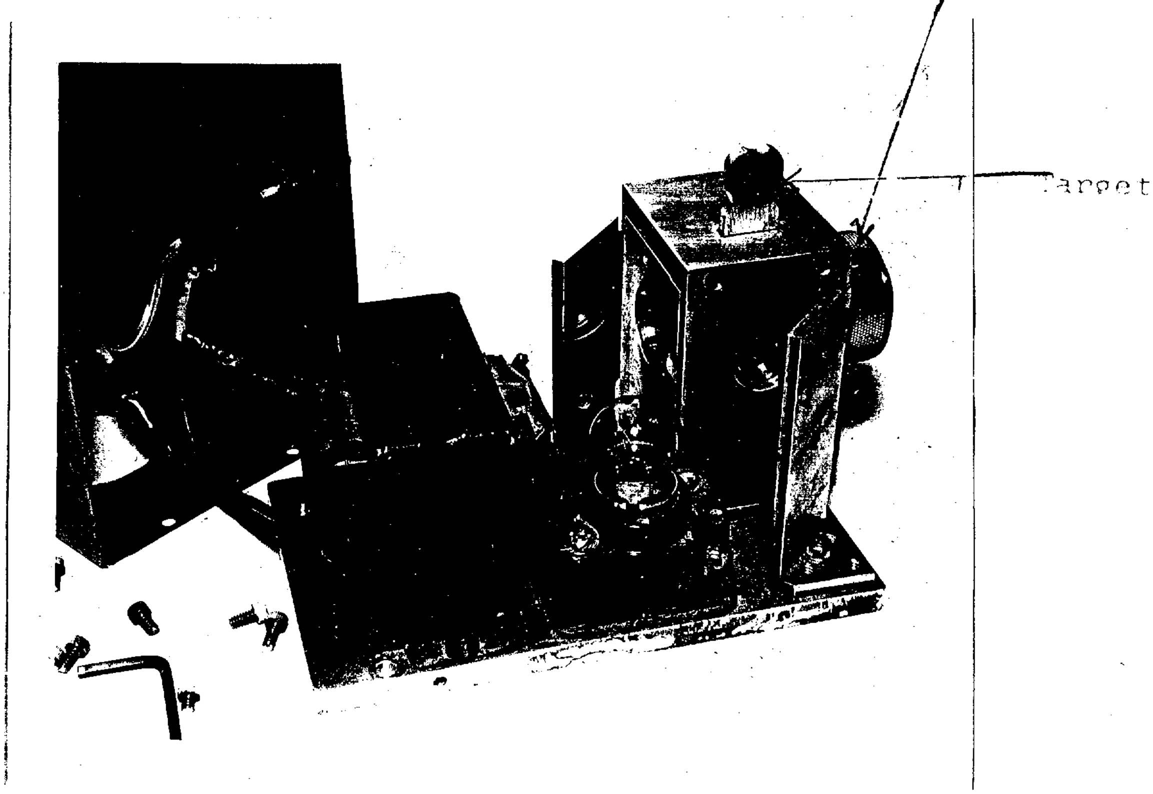
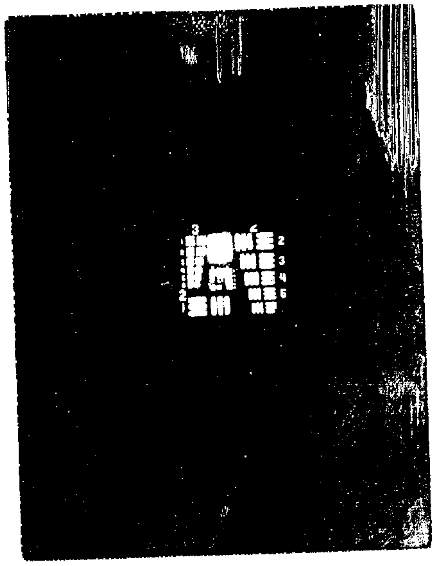
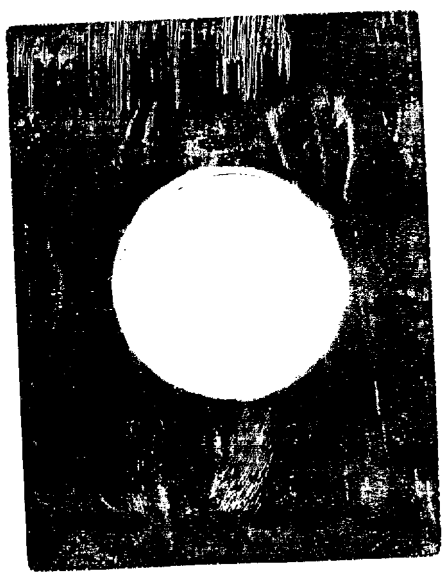


Fig. IV-2



Modulation 1.000



Modulation 0.250

Fig. IV-3

APPENDIX I

Discussion of Tolerances and Measuring Precision of Variable Modulation Targets

The determination of τ as given in equation II-6 is related to the value $1/V_{object}$. Furthermore, we have according to II-5:

$$V_{object} = \lambda V_{target}$$

Therefore the change in λ with rotation of the polarizer is of fundamental importance. This relation is given by

$$\lambda = \cos^2 \alpha . \qquad \text{A-1}$$

In Table A-1 we tabulated α with $\cos^2\alpha$ and $1/\cos^2\alpha$. From this it is seen that below a $V_{object} = 0.1$ (starting with a $V_{target} = 1$) the scale is very crowded. It is therefore advisable to make two sets of targets, one with V=1 and one with V=0.1. This last target will cover the range from $V_{obj.} = 0.1$ to $V_{obj} = 0.01$. This provides all ranges which are desired.

Let us now investigate how good the targets have to be. We shall assume that the maximum transmission of a line in the target is one. The minimum transmission is, however, not zero but β. The maximum transmission in the other beam is γ. If a fraction is transmitted from the target beam and a fraction $1-\lambda$ from the other beam, it is easily shown that the modulation will be:

$$V = \frac{1-\beta}{2\gamma\frac{1-\lambda}{\lambda} + 1 + \beta} \qquad \text{A-2}$$

The normal condition for bar targets is

$$\beta = 0 \qquad \text{A-3}$$

$$\gamma = 1/2$$

α in degrees	$\cos^2\alpha$	$1/\cos^2\alpha$
0	1	1
5	.992	1.008
10	.970	1.031
15	.933	1.072
20	.884	1.131
25	.821	1.218
30	.750	1.333
35	.671	1.490
40	.587	1.704
45	.500	2.000
50	.413	2.421
55	.329	3.040
60	.250	4.000
65	.179	5.587
70	.117	8.547
75	.067	14.925
80	.030	33.333
85	.008	125.000
90	0	

For $\alpha = 72^\circ$ we have $\frac{1}{\cos^2\alpha} = .100$

TABLE I

Under the condition A-3, we find for A-2

$$V = \lambda \qquad \text{A-4}$$

Now if in our target $\beta \neq 0$ we then have:

$$\frac{\partial V}{\partial \beta} = \frac{2\gamma \frac{1-\lambda}{\lambda} + 2\beta}{\left(2\gamma \frac{1-\lambda}{\lambda} + 1 + \beta\right)^2}$$

For our standard conditions, A-3, we have:

$$\frac{\partial V}{\partial \beta} = \lambda(1-\lambda) \qquad \text{A-5}$$

Suppose γ is not adjusted exactly right. We find for the standard conditions:

$$\frac{\partial V}{\partial \gamma} = -2\lambda(1-\lambda) \qquad \text{A-6}$$

Also let us assume λ is not measured exactly right. Now we find for the standard conditions:

$$\frac{\partial V}{\partial \lambda} = -1 \qquad \text{A-7}$$

From A-5, A-6 and A-7 it follows that the quantities β, γ and λ should be made or measured with the same precision with which we want to measure the transfer function, and do not pose severe difficulties for the manufacturing of these parts.

APPENDIX II

STATINTL

Available Computer Programs

[redacted] has an RPC-4000 computer with support personnel. The computer programs available are many, a few of which we will mention here.

Several ray trace programs of our own have special features. All kinds of aspheric, toric and other surfaces can be ray traced. Provisions are available to decenter elements of the system, both laterally and angularly.

In the enclosed article "The Role of Eikonal and Matrix Methods in Contrast Transfer Calculations" our programs to compute the transfer function are described.

Furthermore we have a program which takes a given transfer function and computes the image intensity distribution of a standard Air Force three bar target. The calculations are based on the following formulae:

ν_0 = Object "frequency"

i.e. The reciprocal value of distance of two transparent lines.

ν_l = Limiting frequency of the transfer function.

$A(\nu)$ = Real part of transfer function.

$B(\nu)$ = Imaginary part of transfer function.

X = Coordinate in image.

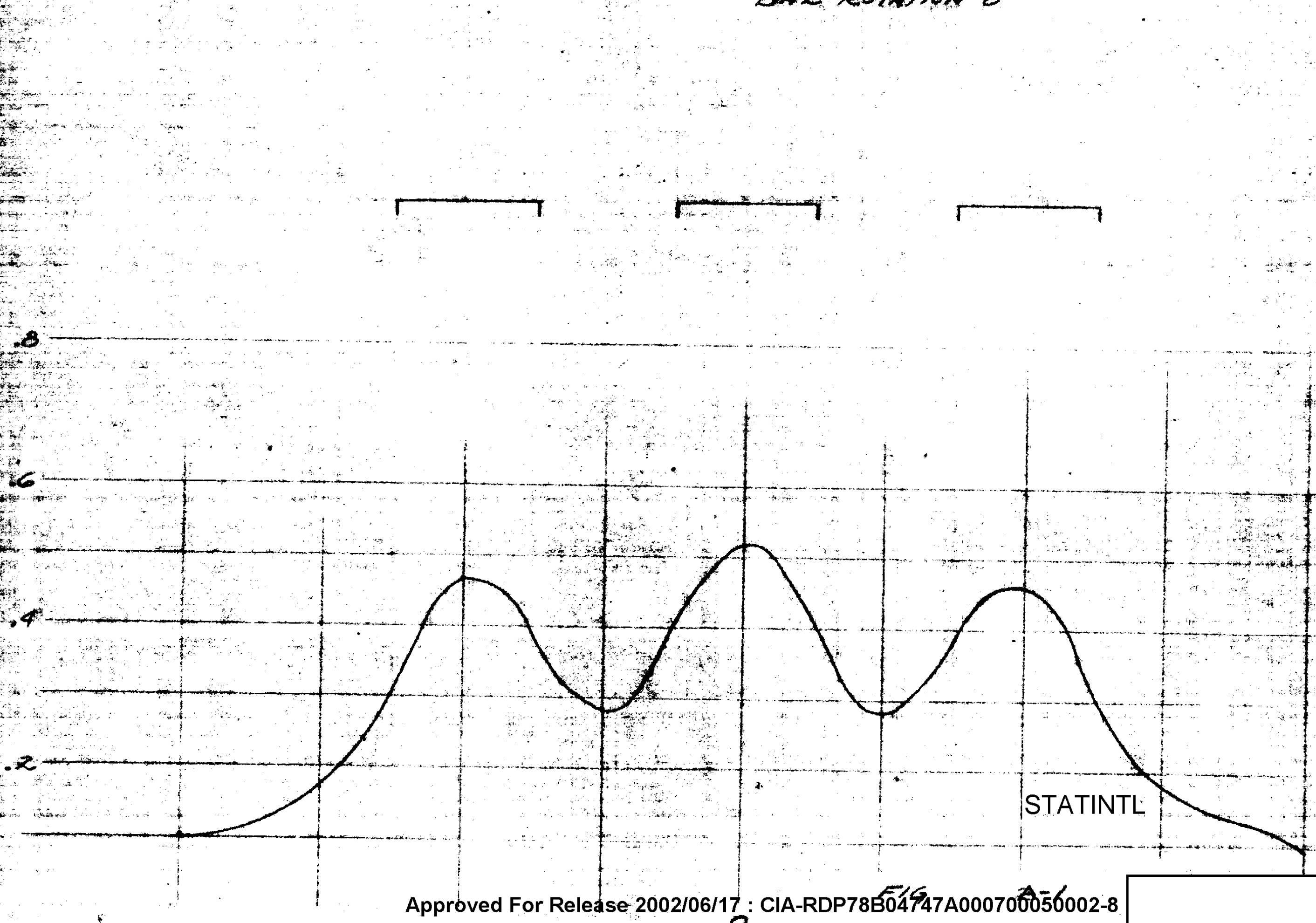
The intensity distribution in the image in the square modulus of:

$$\frac{1}{\nu_0}\int_0^{\nu_1}\Big[A(\nu)\,\mathrm{Cos}\,2\pi\nu x + B(\nu)\,\mathrm{Sin}\,2\pi\nu x\Big]\Big[1+2\,\mathrm{Cos}\,(2\pi\nu/\nu_0)\Big]\frac{\sin(\pi\nu/2\nu_0)}{(\pi\nu/2\nu_0)}\,d\nu$$

Using this expression the integrated intensity in the image equals the integrated intensity in the object, when the object intensity in the transparent parts is unity.

In Figure A-1 we show the intensity distribution in the image of such a target formed by the lens whose transfer function is given in the above article. For our computations we used $\nu_0/\nu_1 = 1/2$ and a line orientation of 0°. In Figures A-2 and A-3 only the orientation of the target is changed to 45° and 90°, respectively.

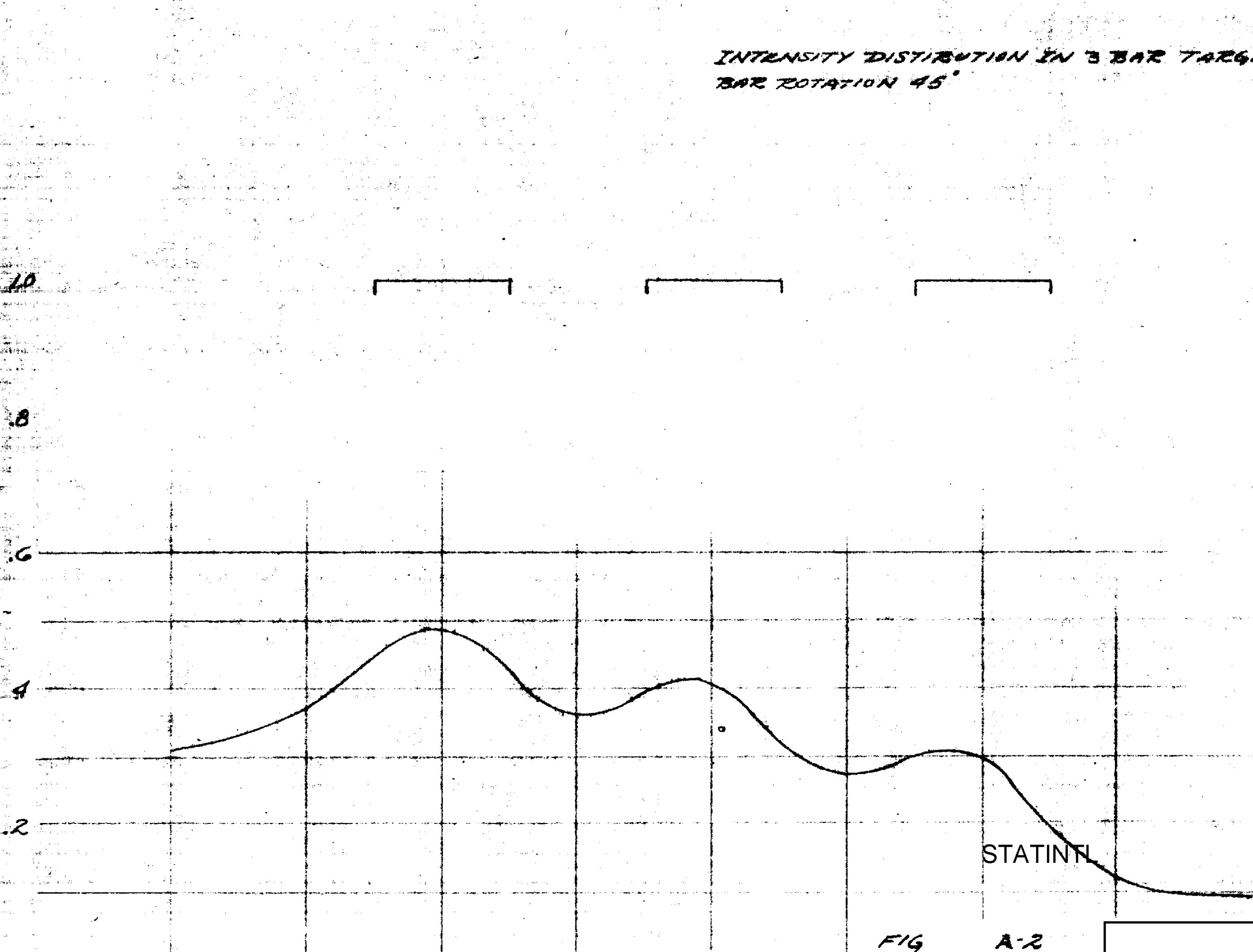
INTENSITY DISTRIBUTION IN 3 BAR TARGET
BAR ROTATION 0°



STATINTL

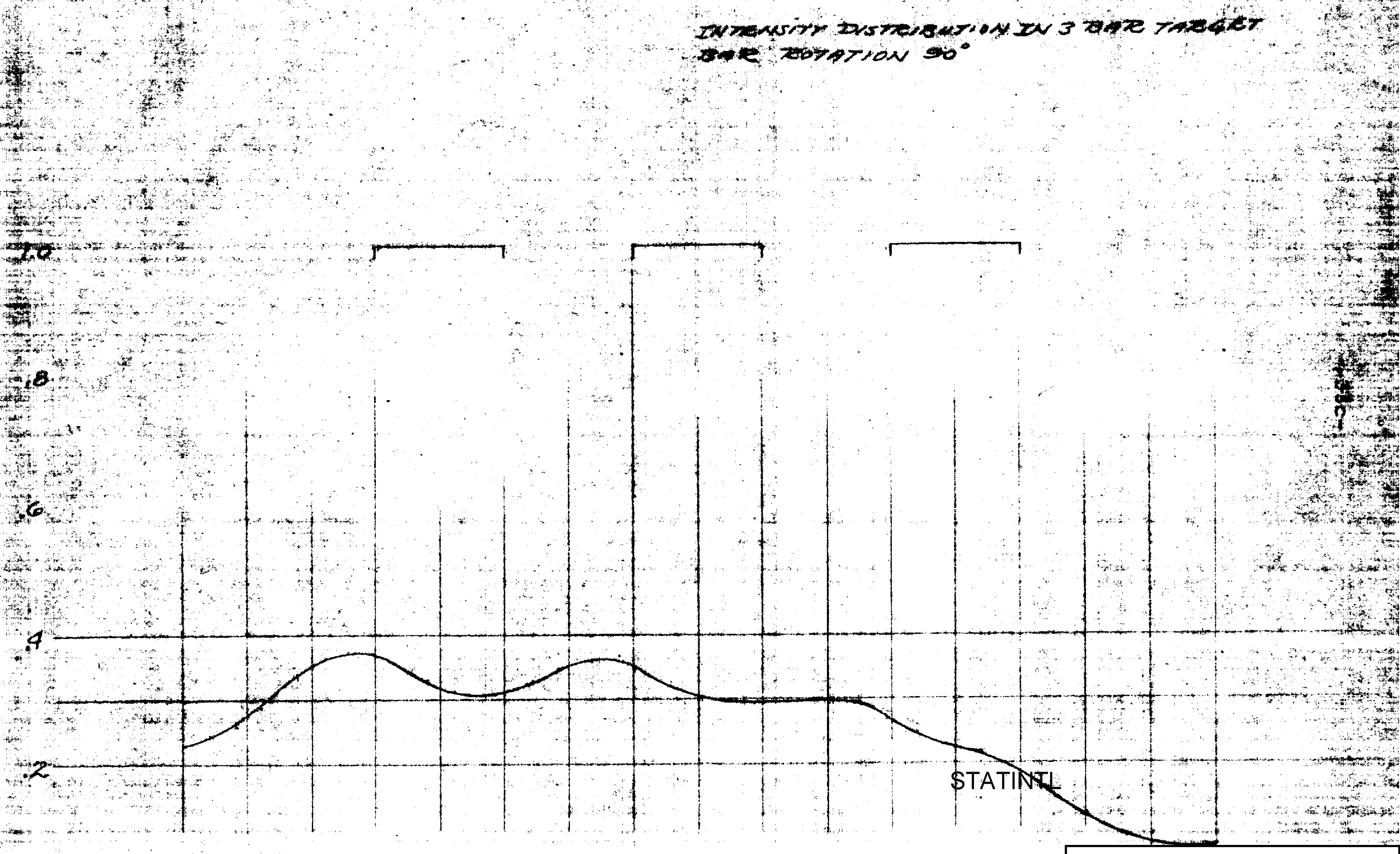
FIG. A-1

INTENSITY DISTRIBUTION IN 3 BAR TARGET
BAR ROTATION 45°



-33b-

FIG A-2



STATINTL

FIG A-3

The Role of Eikonal and Matrix Methods in Contrast Transfer Calculations

W. Brouwer, E. L. O'Neill, and A. Walther

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The Role of Eikonal and Matrix Methods in Contrast Transfer Calculations

W. Brouwer, E. L. O'Neill, and A. Walther

The notion that the optical contrast transfer function is a useful tool for assessing the performance of image-forming instruments has been accepted generally for some time and is now well established. This paper discusses one method of making the transition from ray-trace data to the evaluation of this important function. First, the light distribution in the point image is rigorously derived in terms of an integral over angular coordinates involving the eikonal function about a reference surface at infinity. Then, the ray-trace procedure is developed in the language of refraction and translation matrices culminating in matrix elements which are simply related to the eikonal coefficients of wave optics. Finally, the numerical evaluation of the contrast transfer function in amplitude and phase from these eikonal coefficients is presented, and the paper ends with an example showing the off-axis transfer function for line structures oriented at various azimuths. All calculations are carried out to fifth order in the eikonal coefficients, and emphasis is placed on the usefulness of this approach on relatively slow, low-capacity computing machines.

I. Introduction

The application of Fourier techniques to the theory of image formation has been studied extensively in the preceding decade.

The theory that was developed has been accepted generally as a useful tool in the analysis of optical systems. It centers on two concepts: point-spread function and frequency-transfer function, one being the Fourier transform of the other. Each of these functions can, in principle, be determined when the geometrical aberrations of the lens are known.[1–3]

In practical problems of lens design the ability to evaluate the transfer function numerically would be a great asset to the lens designer. The authors have the impression that one step in the procedure of computing this function is not well known to many workers in the field: the conversion of ray-trace data into wavefront deviations. Following Luneberg[4] and Wolf,[5] we shall treat this problem in a way which is at once simpler and more rigorous, by using eikonal functions instead of wavefront shapes. This approach has the additional advantage that it shows in a unique way the transition from wave optics to geometrical optics.

In the ray-tracing calculations a system of two by two matrices as introduced by Smith[6] and Brouwer[7] will be used to great advantage: the relation between the matrix elements and the eikonal functions will be shown to be very simple, and easier to apply than the usual ray intercepts. This leads to a way of calculating the transfer function that is well suited to relatively slow computers with a rather small memory capacity.

In Sec. II we derive an expression for the point-image amplitude distribution using the eikonal function to describe the aberrations and perform a Fourier transformation over the angular coordinates of a reference surface at infinity. In Sec. III we relate the matrix elements determined from geometrical lens design calculations to the eikonal function. Finally, in Sec. IV, starting with the coefficients in the eikonal expansion, we show several examples of off-axis transfer functions using the numerical integration scheme of Hopkins.[8]

II. Transition from Geometrical Wave Optics

A. Notation

We shall have occasion to use four planes associated with a rotationally symmetrical lens [Fig. 1(a)]. These planes are perpendicular to the axis of the lens and are, respectively, the object plane (coordinates x and y), the entrance pupil plane (coordinates x_1 and y_1), the exit pupil plane (coordinates x_1' and y_1'), and

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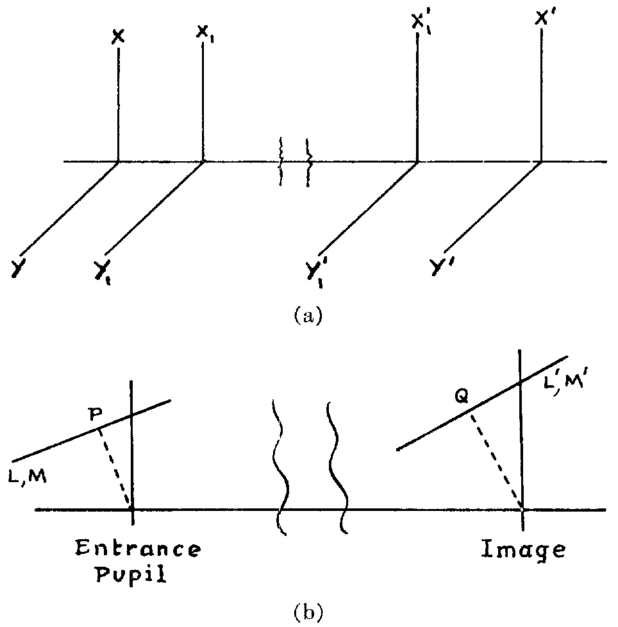

Fig. 1. (a) Coordinate systems. (b) Definition of angle–angle eikonal.

the image plane (coordinates x' and y'). The four axes marked with x lie in one plane, and so do the axes marked with y. The x and y axes are mutually perpendicular and intersect in the axis of the lens, marked by z, z_1, z_1', or z' depending on which of the four planes is used as a reference. The refractive index in the object space is denoted by n; in the image space it is n'. Since we consider only axially symmetric lenses and, unless indicated otherwise, an object point will be understood to lie on the x axis, we are allowed to refer to the x–z plane as the meridional plane.

B. Summary of Fourier Optics

In order to establish our notation and for reference purposes we shall give a brief summary of the basic concepts in Fourier optics. In what follows, $F(x_1',y_1')$ describes the complex scalar disturbance over the exit pupil plane, $a(x',y')$ the complex amplitude distribution in the image of a point source, $s(x';y')$ the intensity-spread function, and finally $D(\nu_x,\nu_y)$ the frequency response or transfer function for the system. By a direct application of Huygen's principle together with approximations that are quite valid for most optical systems it is not difficult to show that $a(x',y')$ and $F(x_1',y_1')$ are Fourier transform pairs in the form

$$a(x', y') = \int\int_{-\infty}^{+\infty} F(\nu_x, \nu_y) \exp [2\pi i(\nu_x x' + \nu_y y')]\, d\nu_x d\nu_y, \qquad (1)$$

where ν_x and ν_y are reduced coordinates in the exit pupil about which we shall say more later, and where all constants and the finite area of integration have been absorbed into $F(\nu_x,\nu_y)$. Now by virtue of the fact that we are treating the optical system as a filter of spatial frequencies there exists a further Fourier transformation between $s(x',y')$ and $D(\nu_x,\nu_y)$ in the form

$$D(\nu_x, \nu_y) = \frac{\int\int_{-\infty}^{+\infty} s(x', y') \exp [-2\pi i(\nu_x x' + \nu_y y')] dx'dy'}{\int\int_{-\infty}^{+\infty} s(x', y')\, dx'dy'}, \qquad (2)$$

where the denominator has been introduced for normalization purposes ($|D(0)| = 1$). Capitalizing both on the fact that $s(x',y') = |a(x',y')|^2$ and the convolution theorem for the transform of a product, we end up with a direct relation between the disturbance over the exit pupil and the frequency response in terms of the well-known integral

$$D(\nu_x, \nu_y) = \frac{\int\int_{-\infty}^{+\infty} F(\mu_x, \mu_y) F^*(\mu_x - \nu_x, \mu_y - \nu_y) d\mu_x d\mu_y}{\int\int_{-\infty}^{+\infty} |F(\mu_x, \mu_y)|^2 d\mu_x d\mu_y}. \qquad (3)$$

These relations together with their physical interpretations have been fully described in the literature, and the uninitiated reader is invited to consult the references for further details of the Fourier approach (e.g., refs. 2 and 3). As it stands, the phase portion of $F(\mu_x, \mu_y)$ describes a surface of constant phase about a reference sphere passing through the center of the exit pupil whose center is an appropriate point in the image plane. Unfortunately, we have not found this a convenient reference surface in our attempts to bridge the gap between wave optics and geometrical optics. This transition is an important point. We wish to emphasize that the shape of the wavefront can be determined to *any accuracy desired* by means of the laws of geometrical optics. Consequently, any approximation in the translation of these geometrical data to data used in the work on diffraction involves an unnecessary waste of available information. Methods have been suggested to remove the wavefront deformation function from the exponent of the diffraction integral in an artificial manner so as to use quasi-geometrical methods in passing from the exit pupil to the image plane. These techniques (e.g., spot diagrams) must be treated with utmost care; in this paper we shall not take recourse to these approximations.

C. Wavefront Deviations

Our first aim shall be to establish a reasonably exact derivation for Eq. (1), which will automatically lead to the proper definition of ν_x and ν_y. We shall, as is usual in this field, restrict ourselves to the Huygens–Fresnel diffraction theory. There are several undesirable features in the usual derivation of Eq. (1). We mention:

(1) The shape of the wavefront depends on where one choses its location.

(2) Some authors measure the deviation of the wavefront along the rays. Others measure along the radii of the reference sphere.

(3) The relations between the coordinates that define a ray geometrically and the coordinates suited to diffraction calculations are very complicated.

(4) The diffraction integral takes only approximately the form of a Fourier integral.

The approach taken to this problem in the present paper, found to be due to Luneberg,[4] and also treated by Wolf[5] avoids these problems.

In Fig. 2, let $P(x,y)$ be an object point of which an optical system creates a diffraction image in the (not necessarily Gaussian) image plane (x',y'). Let $P'(x_0' + \xi', y_0' + \eta')$ be a point in which we wish to evaluate the amplitude of the diffracted light; ξ' and η' are coordinates in the diffraction pattern, measured in the image plane with respect to a reference point (x_0', y_0') which conventionally is chosen as the intersection point of the principal ray with the image plane.

When the light has traversed the optical system, the pencil of light is completely determined by a wavefront Σ. The line AQ represents a ray in the image space.

The wavefront being a surface of equal phase, the amplitude in the point P' is proportional to:

$$a(P') = \int_\Sigma F(A) \exp\left(\frac{2\pi i}{\lambda} W\right) d\sigma, \tag{4}$$

in which $W = \overline{AP'}$ and $F(A)$ is the amplitude distribution over the wavefront Σ. A discussion of this amplitude distribution in the wavefront is outside the scope of this paper.

Let the direction of rays in the image space be given in terms of their optical direction cosines (L',M',N') which are defined as the geometrical direction cosines multiplied with the refractive index of the image space. Let OS be the normal drawn from the origin in the (x',y') plane onto the ray AQ. Then the optical path length $\overline{PAS}$ considered as a function $E(x,y,L',M')$ of x,y,L', and M' is known as the (point-angle) mixed eikonal of the system.[9,10]

When this function is known, the coordinates (x',y') of the intersection point of a ray with the image plane are given by:

$$\frac{\partial E}{\partial L'} = -x', \; \frac{\partial E}{\partial M'} = -y', \tag{5}$$

and the direction cosines of the rays in the object space:

$$\frac{\partial E}{\partial x} = -L, \; \frac{\partial E}{\partial y} = -M.$$

Let RP' be the normal from P' drawn onto the ray AQ. Then we can write for the path length $\overline{AP'}$ in Eq. (4):

$$\overline{AP'} = E(L', M') - \overline{PA} + \overline{SR} + (\overline{AP'} - \overline{AR}), \tag{6}$$

in which the dependence of E on x and y is omitted because we assume the object point to be fixed. The path $\overline{PA}$ is constant and may accordingly be dropped. The location of the wavefront is irrelevant, as long as we do not choose it "too close to the image plane." A great simplification is obtained if we make full use of this freedom and move the wavefront out to infinity. In that case the term $(\overline{AP'} - \overline{AR})$ in (6) reduces to zero and the line AP' becomes parallel to the ray AQ. For the projection $\overline{SR}$ of the line OP' onto the ray AQ we can write:

$$\overline{SR} = L'(x_0' + \xi') + M'(y_0' + \eta');$$

consequently, we have:

$$W = E(L,' M') + L'x_0' + M'y_0' + \xi'L' + \eta'M'.$$

A point on the wavefront is now no longer specified by linear coordinates; it must be specified as a direction. So the diffraction integral (1) reduces to:

$$a(P') = \iint_\Sigma F(L'M') \exp \frac{2\pi i}{\lambda} [W_0(L',M') + \xi'L' + \eta'M']dL'dM' \tag{7}$$

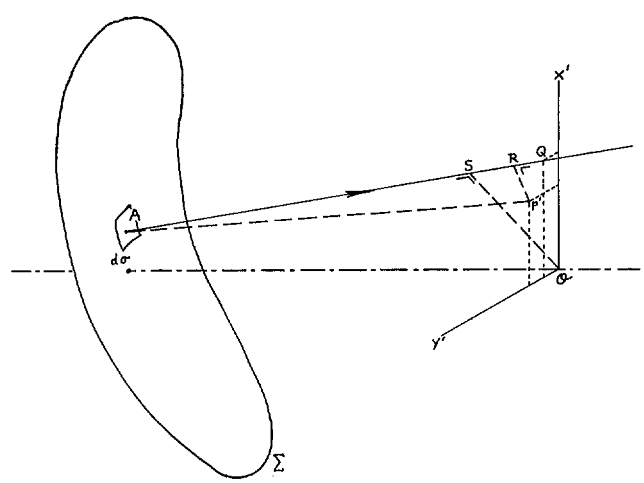

Fig. 2. Wavefront configuration.

in which

$$W_0(L,'M') = E(L,'M') + L'x_0 + M'y_0, \tag{8}$$

a function which is open to easy numerical evaluation, as will be shown subsequently.

Comparing Eqs. (7) and (8) with Eq. (1) we observe first of all that the rather vague concept of wavefront deformation has been replaced by a well-defined eikonal function. We also notice that (7) represents a Fourier transformation, provided that we use as variables in the frequency domain:

$$\begin{aligned} \nu_x &= L'/\lambda, \\ \nu_y &= M'/\lambda. \end{aligned} \tag{9}$$

Every pair of values for ν_x and ν_y defines uniquely a point in the exit pupil. The integration over the exit pupil coordinates is now replaced by an integration over direction cosines in the image space. These direction cosines are a natural product of the ray trace, and, consequently, one does not need the linear exit pupil coordinates at all. Luneberg[4] has shown that the limiting procedure of moving the wavefront to infinity is an essential step in the electromagnetic theory of image formation (see also Wolf[5]).

The amplitude function $\bar{F}(L',M')$ is still unknown and must be determined by external means. It may often be assumed to be constant. (See, however, refs. 11 and 12, in which an object point is assumed that radiates uniformly in all directions.)

Equations (7) and (8) form an ideal bridge between geometrical optics and wave optics. Equation (7) shows that the diffraction integral may be considered as a Fourier transform, even for wide apertures and large field angles. Neither an artificially tipped image plane nor a troublesome reference sphere need be introduced. Equation (7) shows a close relationship between the eikonal functions and diffraction theory. This relationship becomes even more apparent when we apply the method of stationary phase to Eq. (7). For large aberrations, the direction (L',M') contributing most to the amplitude in the point $(x_0' + \xi',y_0'+\eta')$ is found by requiring that the exponent be stationary with respect to L' and M'. This yields:

$$\frac{\partial E}{\partial L'} + x_0' + \xi' = 0,$$

$$\frac{\partial E}{\partial M'} + y_0' + \eta' = 0,$$

which are the exact geometrical relations to be expected. We see that in the diffraction theory of image formation all of geometrical optics remains valid; it is however, elevated into the exponent of the diffraction integral.

III. Numerical Evaluation of the Eikonal

The wave-optics description completed, we turn now to the calculation of the eikonal function. In the conventional ray-tracing procedure it is not feasible to determine the relation between the heights of intersection in the image plane and the pupil coordinates in a closed form. We must sample this function numerically, and, if we wish, then determine intermediate values by interpolation. For the eikonal function we have to apply the same technique, for the same reason. This could be done by path-length computations along the rays traced; however, to attain the desired accuracy a double word-length computation has to be used. With moderately small computers this becomes time-consuming. One can, however, simplify these computations considerably by applying the interpolation not to the eikonal function itself but to its first derivatives with respect to the rotationally invariant variables. In a power series development of the eikonal function a term of a degree n in the linear variables leads to terms of a degree $n - 2$ in the power series of the above-defined first derivative, whence the greater accuracy obtainable. Furthermore, these first derivatives are directly related to matrix elements (obtained from a ray trace), that we shall define presently.

A ray-trace procedure consists in following a ray through an optical system. There are fundamentally two steps involved. At each refracting surface the ray changes direction, and in going from one surface to the next the intersection point of the ray with this next surface has to be found. Let us first describe the refraction of a ray at the ith surface of the system. The coordinates of the point of intersection of the ray with the surface in the space before refraction are denoted by x_i, y_i and by x_i' and y_i' when considered in the space after refraction. The optical direction cosines in these two spaces are denoted by L_i, M_i and L_i' and M_i'. It has been shown that the refraction can be written in the form (see ref. 7):

$$\begin{bmatrix} L_i' \\ x_i' \end{bmatrix} = \begin{bmatrix} 1 & -A_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} L_i \\ x_i \end{bmatrix} = R_i \begin{bmatrix} L_i \\ x_i \end{bmatrix}$$

and

$$\begin{bmatrix} M_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} 1 & -A_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} M_i \\ y_i \end{bmatrix} = R_i \begin{bmatrix} M_i \\ y_i \end{bmatrix}, \tag{10}$$

where

$$A_i = \frac{n_i' \cos\varphi_i' - n_i \cos\varphi_i}{r_i}.$$

n and n' are the refractive indices of the media in front and behind the surface, φ_i and φ_i' are the angles of incidence and refraction, and r_i is the subnormal of the refracting surface for the ray considered. For a spherical surface this is the radius itself.

Now, with a similar notation, we can write for the translation to the next surface:

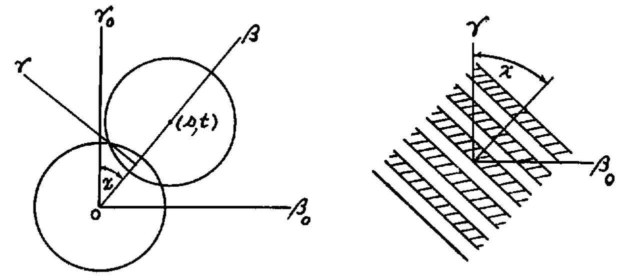

Fig. 3. Rotation of coordinate axes for the computation of the contrast transfer function.

$$\begin{bmatrix} L_{i+1} \\ x_{i+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ T_i' & 1 \end{bmatrix} \begin{bmatrix} L_i' \\ x_i' \end{bmatrix} = T \begin{bmatrix} L_i' \\ x_i' \end{bmatrix}$$

and (11)

$$\begin{bmatrix} M_{i+1} \\ y_{i+1} \end{bmatrix} = \begin{bmatrix} L & 0 \\ T_i' & 1 \end{bmatrix} \begin{bmatrix} M_i' \\ y_i' \end{bmatrix} = T_i \begin{bmatrix} M_i' \\ y_i' \end{bmatrix},$$

where

$$T_i' = t_i'/n_i.$$

t_i' is the distance measured along the ray between the points x_i', y_i' and x_{i+1}, y_{i+1}. It should be noted that the ray between object point and point of incidence on the first surface can be described by a matrix of the form T and will be denoted by T_0. In the same way in the image space we have a matrix T_n giving the coordinates of the ray in the image plane in terms of the coordinates of the ray at the last surface k. When all matrices R_i and T_i are computed the ray coordinates in the image plane in terms of the coordinates of the ray in the object plane can be found by

$$\begin{bmatrix} L' \\ x' \end{bmatrix} = T_k R_k T_{k-1} R_{k-1} \ldots T_2 R_2 T_1 R_1 T_0 \begin{bmatrix} L \\ x \end{bmatrix},$$

and similarly for M' and y' as a function of M and y. Since both T and R are 2×2 matrices, the result of the matrix multiplications will be a 2×2 matrix of the form:

$$\begin{bmatrix} L' \\ x' \end{bmatrix} = \begin{bmatrix} B & -A \\ -D & C \end{bmatrix} \begin{bmatrix} L \\ x \end{bmatrix} \text{ and } \begin{bmatrix} M' \\ y' \end{bmatrix} = \begin{bmatrix} B & -A \\ -D & C \end{bmatrix} \begin{bmatrix} M \\ y \end{bmatrix}. \tag{12}$$

Note that the final "x" and "y" matrices are the same; this is due to the rotational symmetry.

Since the optical systems considered here have an axis of symmetry, we can achieve a simplification by introducing the following rotationally invariant quantities:

$$\begin{aligned} u_1 &= \tfrac{1}{2}(x^2 + y^2), \\ u_2 &= L'x + M'y, \\ u_3 &= \tfrac{1}{2}(L'^2 + M'^2). \end{aligned} \tag{13}$$

Introducing (13) and (5) into Eq. (12) yields

$$\left(-B\frac{\partial E}{\partial u_1} - A\right)x - \left(B\frac{\partial E}{\partial u_2} + 1\right)L' = 0,$$

$$\left(D\frac{\partial E}{\partial u_1} + C + \frac{\partial E}{\partial u_2}\right)x + \left(D\frac{\partial E}{\partial u_2} + \frac{\partial E}{\partial u_3}\right)L' = 0.$$

Since these relations are valid for all values of x and L' each coefficient should be zero; yielding:

$$\begin{aligned} \frac{\partial E}{\partial u_1} &= -\frac{A}{B}, \\ \frac{\partial E}{\partial u_2} &= -\frac{1}{B}, \\ \frac{\partial E}{\partial u_3} &= \frac{D}{B}, \\ BC - AD &= +1. \end{aligned} \tag{14}$$

Numerical values for these partial derivatives of the eikonal function are thus easily obtained from ray traces. A simple integration will then give the function E.

There are many ways in which this calculation can be performed. In the following a procedure is worked out using truncated power series up to and including the sixth order in the aperture variables to find E as a function of u_2 and u_3, and thus as a function of the coordinates L' and M' that we wish to use in the work on diffraction.

Since we are interested only in the function E for one object point, u_1 is a constant and E is a function of u_2 and u_3 only. Let us assume the form

$$\begin{aligned} E(u_2, u_3) &= \Sigma_i \Sigma_j E_{ij} u_2{}^i u_3{}^j \\ &\approx E_{00} + E_{10}u_2 + E_{01}u_3 \\ &\quad + E_{20}u_2{}^2 + E_{11}u_2u_3 + E_{02}u_3{}^2 \\ &\quad + E_{30}u_2{}^3 + E_{21}u_2{}^2u_3 + E_{12}u_2u_3{}^2 + E_{03}u_3{}^3 \\ &\quad + \ldots. \end{aligned} \tag{15}$$

From this it follows that

$$\begin{aligned} \frac{\partial E}{\partial u_2} &= -\frac{1}{B} = E_{10} + 2E_{20}u_2 + E_{11}u_3 + 3E_{30}u_2{}^2 + 2E_{21}u_2u_3 \\ &\qquad + E_{12}u_3{}^2 + \ldots, \\ \frac{\partial E}{\partial u_3} &= \frac{D}{B} = E_{01} + E_{11}u_2 + 2E_{02}u_3 + E_{21}u_2{}^2 + 2E_{12}u_2u_3 \\ &\qquad + 3E_{03}u_3{}^2 + \ldots. \end{aligned} \tag{16}$$

All coefficients appearing in the desired function E appear in the functions (16) except E_{00}. However, E_{00} is a constant and therefore not needed in the computations on diffraction. The coefficients E_{ij} in (16) can now be found from the values of the matrix elements B and D for five or more rays. We use a least-square method where the number of rays used is determined by the desired fit.

It is interesting to note that in this approximation only five meridional rays are needed. The object is situated on the x axis, and so we have for meridional rays:

$$u_2 = L'x,$$
$$u_3 = {}^1/_2\, L'^2,$$

and thus:

$$-\frac{1}{B} = E_{10} + (2E_{20}x)L' + ({}^1/_2E_{11} + 3E_{30}x^2)\, L'^2 + (E_{21}x)L'^3 + {}^1/_4E_{12}L'^4,$$

$$\frac{D}{B} = E_{01} + (E_{11}x)L' + (E_{02} + E_{21}x^2)L'^2 + (E_{12}x)L'^3 + {}^3/_4E_{03}L'^4.$$

Five meridional rays, using the first equation gives: E_{10}, E_{20}, $({}^1/_2E_{11} + 3E_{30}\, x^2)$, E_{21}, and E_{12}. Four of the same rays, and the second equation gives: E_{01}, E_1, $(E_{02} + E_{21}\, x^2)$, E_{03}. Simple inspection shows that all coefficients can then be computed. When the object is at infinity the same procedures can be used with the help of the angle–angle eikonal as shown in the appendix.

IV. Calculation of the Transfer Function

Having shown the relation between the information supplied by the ray trace (the matrix elements) and the coefficients E_{ij} of the eikonal function we now proceed to calculate the transfer function. In doing so it proves convenient to define normalized variables over which we carry out the numerical integration. Letting L_m' and M_m' represent the maximum* direction cosines as seen from the image field point we now define

$$\beta_0 = L'/L'_m \text{ and } \gamma_0 = M'/M_m'.$$

Further, we define normalized polar coordinates $\beta_0 = \rho\cos\phi$; $\gamma_0 = \rho\sin\phi$ such that $0 \leq \rho \leq 1$ and we see that aside from scale factors which can be absorbed into the coefficients, u_2 can be replaced by β_0 and u_3 by ρ^2. We now can write the basic integral for the evaluation of the transfer function in terms of these normalized variables as:

$$D(s,t) = \frac{\int\int_{-\infty}^{\infty} F(\beta_0, \gamma_0)F^*(\beta_0 - s, \gamma_0 - t)\, d\beta_0 d\gamma_0}{\int\int_{-\infty}^{\infty} |F_0(\beta_0, \gamma_0)|^2\, d\beta_0 d\gamma_0}, \tag{17}$$

where s and t are line frequencies normalized such that they run from 0 to 1, and where for our purposes we take

$$F(\beta_0, \gamma_0) = \exp\left[\frac{2\pi i}{\lambda}E(\beta_0, \gamma_0)\right], \quad \beta_0^2 + \gamma_0^2 \leq 1$$
$$= 0, \qquad \beta_0^2 + \gamma_0^2 > 1.$$

Finally, for ease in computation it turns out to be convenient to perform a translation and rotation of axes to β, γ centered at the common area of the displaced circles* such that β points along the direction of the normal (ψ) of the line structure in the object plane (see Fig. 3). With these changes of variables our basic integral can be cast into the form:

$$D(s,\psi) = \frac{\int\int_{-\infty}^{\infty} F\left(\beta + \frac{s}{2}, \gamma\right)F^*\left(\beta - \frac{s}{2}, \gamma\right)d\beta d\gamma}{\int\int_{-\infty}^{\infty} |F(\beta, \gamma)|^2 d\beta d\gamma}. \tag{18}$$

At this point we adopt the numerical integration scheme of Hopkins[8] and write first

$$D(s,\psi) = \frac{1}{a}\int\int_{\infty} \exp\,[iksV(\beta, \gamma;\ \psi)]\ d\beta d\gamma, \tag{19}$$

where $a = \int\int d\beta d\gamma$ is the normalization constant and $V(\beta, \gamma; \psi)$ is given by:

$$V(\beta, \alpha;\ \psi) = \frac{1}{s}\left[E\left(\beta + \frac{s}{2}, \gamma;\ \psi\right) - E\left(\beta - \frac{s}{2}, \gamma;\ \psi\right)\right] \tag{20}$$
$$= \frac{\partial E}{\partial\beta} + \frac{1}{3!}\left(\frac{s}{2}\right)^2\frac{\partial^3E}{\partial\beta^3} + \frac{1}{5!}\left(\frac{s}{2}\right)^4\frac{\partial^5E}{\partial\beta^5}.$$

Now as Hopkins[8] has demonstrated (see also Marchand and Phillips[13]) this integral can be approximated by a double summation taken over all the elementary cells (ϵ_x, ϵ_y) that fall within the common area in the form

$$D(s,\psi) = \frac{1}{N}\,\Sigma_m\Sigma_n e^{iZ}\,\frac{\sin X}{X}\,\frac{\sin Y}{Y}, \tag{21}$$

where $N = a/(4\epsilon_x\epsilon_y)$ is the number of rectangles of area $4\epsilon_x\epsilon_y$ that fall within the full area a and where

$$X = \epsilon_x ks\frac{\partial V}{\partial\beta},$$
$$Y = \epsilon_y ks\frac{\partial V}{\partial\alpha}, \tag{22}$$
$$Z = ksV(\beta, \gamma;\ \psi),$$

each of which is evaluated at the center of the elementary cells. Finally, to complete this description we note that we can also write $D(s, \psi)$ in the form

$$D(s,\psi) = |D(s,\psi)|e^{i\theta(s,\psi)}, \tag{23}$$

* Actually L_m' and M_m' are symmetrized forms of the direction cosines that limiting rays make with the x', y' axes as seen from an off-axis field point.

* In actual practice as one gets off-axis the region of integration is the common area of two displaced ellipses. Further, the effect of vignetting must be taken into consideration.

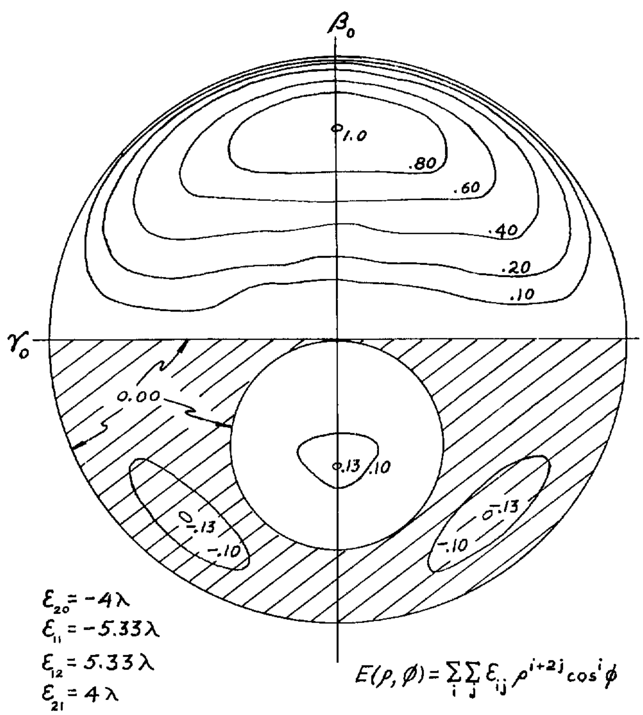

Fig. 4. Rough sketch of normalized wave deformation.

where

$$|D(s, \psi)|^2 = D_R^2(s, \psi) + D_I^2(s, \psi),$$

$$\theta(s, \psi) = \tan^{-1} D_I/D_R,$$

and where

$$D_R(s, \psi) = \frac{1}{N} \Sigma_m \Sigma_n \cos Z \frac{\sin X}{X} \frac{\sin Y}{Y},$$

$$D_I(s, \psi) = \frac{1}{N} \Sigma_m \Sigma_n \sin Z \frac{\sin X}{X} \frac{\sin Y}{Y}.$$

It is to be noted in this approach that X, Y, and Z depend upon $V(\beta,\gamma;\ \psi)$ which in turn depends upon $E(\beta,\ \alpha;\ \psi)$. We have found it convenient to express $E(\beta, \alpha; \psi)$ in the form

$$E(\beta,\ \alpha;\ \psi) = \sum_{k,l}^{k+l \leq 6} \sum_{=0} A_{kl}(\psi) \beta^k \gamma^l, \tag{24}$$

where the original eikonal coefficients and the orientation of the line structure have been absorbed into the 49 matrix elements A_{kl} (of which 25 are zero in fifth-order theory). As Marchand and Phillips have pointed out, the extension of Hopkins' method to fifth order is a straightforward problem in numerical integration. It is our experience also that a cell size of 0.05 units is sufficiently accurate for all purposes encountered in practice. Moreover, this approach does not necessarily require a high-speed, large-capacity computer. For our purposes we find the RPC 4000, a relatively modest computer, perfectly adequate. A complete curve, yielding 20 equally spaced points for the amplitude and 20 points for the phase takes $4^1/_2$ hr. Often the amplitude falls off and remains below a physically meaningful value before the run is completed, and the computer is instructed to proceed directly to the next case of physical interest. These remarks are not meant to minimize the advantages of high-speed computers in optical design. The point we wish to make here is simply that the transfer function is not an end in itself but merely the most useful tool we have to date to assess the relative merits of one design over another. As such this function becomes useful only during the final stages of design after earlier attempts have been rejected for practical and other reasons. Having reached the point where one wishes now to concentrate on a few critical cases, the assessment using the contrast transfer function comes into play, and we here merely wish to emphasize that such evaluations are well within the scope of the more modest computers.

We should like to close this section by giving a numerical illustration for a wave deformation often encountered in practice. Figure 4 shows a wave deformation exhibiting a typical "bubble" along the meridional section as seen from an off-axis field point. On axis all coefficients are zero, and the now familiar transfer function for this case is shown in Fig. 5. Off axis the situation is different, and we show in Fig. 5 the amplitude and phase of the contrast transfer function for line structures whose normals point along the sagittal and meridional section and at 45° to these. Needless to say we find such curves quite useful in evaluating the performance of optical systems.

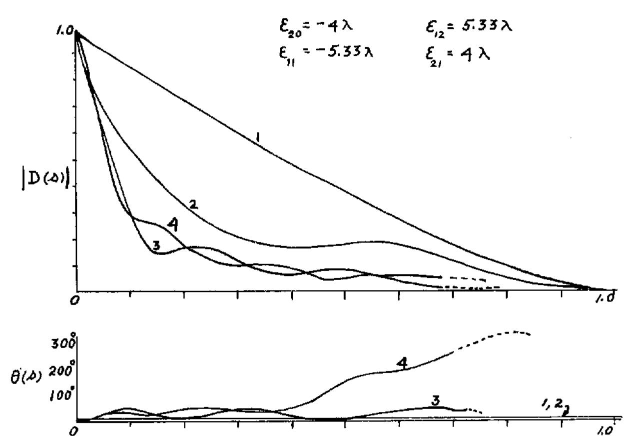

Fig. 5. Contrast transfer functions for wave deformation shown in Fig. 4 at several azimuth angles. Curve 1—on-axis; curve 2—off-axis $\psi = 0$; curve 3—off-axis $\psi = \pi/4$; curve 4—off-axis $\psi = \pi/2$.

We wish to express our gratitude to S. Daniels and H. Sijgers for their roles in bringing this paper to completion.

Appendix

In the case that the object is at infinity it is convenient to use a slightly different eikonal function, the so called angle–angle eikonal.

From the center of the entrance pupil we construct a perpendicular to the ray considered. (See Fig. 1.) The same is done in the object plane. The eikonal is now the optical path length P between the footpoints of these perpendiculars and is a function of L, L', M, and M'.

For a given point at infinity L and M are constants and all points P for rays from this object point are on one wavefront. For this eikonal the following relations hold (see ref. 9)

$$x = \frac{\partial E,}{\partial L} \quad -x' = \frac{\partial E}{\partial L'},$$

$$y = \frac{\partial E,}{\partial M} \quad -y' = \frac{\partial E}{\partial M'}.$$

For rotational symmetric systems the following rotationally invariant quantities are introduced

$$\bar{u}_1 = {}^1/_2(L^2 + M^2),$$

$$\bar{u}_2 = LL' + MM',$$

$$\bar{u}_3 = {}^1/_2(L'^2 + M'^2).$$

From this it is easily derived that

$$\frac{\partial E}{\partial \bar{u}_1} = \frac{B}{A},$$

$$\frac{\partial E}{\partial \bar{u}_2} = -\frac{1}{A},$$

$$\frac{\partial E}{\partial \bar{u}_3} = \frac{C}{A},$$

$$BC - AD = +1.$$

All other computations are the same as in the case with a finite object distance.

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V. Proposed Statement of Work

We propose a two-phase program to achieve the construction of a field sine wave tester.

Phase 1: During this period we propose to do the following:

a) Design investigation to define best possible optical layout.

b) Procurement or fabrication of the necessary filters and optics.

c) Theoretical investigation of influence of target imperfections, and possible improvements of measurement techniques.

d) Fabrication of breadboard model to verify designs.

e) Technical proposal for fabrication model for Phase II

The deliverable items during this phase will be:

I) Monthly progress reports

II) Final report

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III) Demonstration at [redacted] of breadboard model and other setups developed during this phase.

A CPFF price proposal for this phase is included.

Delivery: Investigation study to be completed nine months after receipt of order.

Phase II: Fabrication of final instrument (or instruments). With the instruments we will delivery instruction manuals for the use of this equipment.

Since this phase can not be well defined before the end of Phase I, it is difficult to state a definite price. For budgetary purposes we think that such an instrument will cost about each. We plan to include in Phase I, Item e, a fixed price quotation for Phase II. STATINTL

Delivery: First unit to be delivered three months after agreement has been reached on breadboard model and after receipt of purchase order, whichever is the later date.

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STANDARD CONDITIONS OF SALE

BAILED PROPERTY

In accordance with the practice of the industry with respect to stock furnished by the customer, [redacted] ("the Company") does not accept any liability for the destruction of bailed property.

CANCELLATION

This contract may be cancelled by the Purchaser only upon the payment of reasonable cancellation charges which shall take into account expenses already incurred for labor and material costs and overhead and other commitments made by the Company. Filing of a petition in bankruptcy or commencement of any insolvency proceeding pursuant to State law shall be deemed a cancellation by the Purchaser.

In cases where the Purchaser has paid part of the cost of the equipment before delivery and the Company is unable to manufacture and deliver equipment in accordance with the specifications within a reasonable period beyond the shipping estimate, the Company agrees to return all payments by the Purchaser and each agrees to terminate this contract without further liability to the other.

In no event shall any claim for consequential damages be made by either party.

CONTRACT

Except as otherwise agreed by the Parties in writing, this contract shall be final upon the acceptance by the Purchaser of the Statement of Work proposed in the technical quotation. The terms contained herein shall constitute the entire contract and shall not be modified by standard clauses in the customer's purchase order or elsewhere unless the same shall have been specifically denoted and acknowledged by the Company; provided however, that this contract shall incorporate by reference such terms and conditions as are required by law and Government regulations responsive to requests to bid by agencies and prime contractors of the United States. In case of conflict between these Standard Conditions and such laws and regulations the latter shall prevail. The Purchaser shall not assign this contract without the written consent of the Company.

DELIVERY

Unless otherwise specified all quotations are f.o.b. [redacted] If the customer fails to provide portage an additional charge will be made for shipping and insurance. Adherence to shipping dates depends upon prompt receipt of necessary information and materials. If "the Company" fails to receive said information and materials, for reasons under the control of the customer, it reserves the option either of delivering substantially completed equipment as scheduled, or delaying delivery at the customer's expense until receipt of same. If shipment is delayed by the Purchaser, payments shall become due from date the Company is prepared to make shipment.

The Company shall not be liable for failure to deliver resulting from causes beyond its control, such as acts of God, acts of the Purchaser, negligence of processors, acts of civil or military authorities, strikes, fires, pestilence, riots, inability due to causes beyond its reasonable control to obtain necessary labor, materials or manufacturing facilities. In the event of delay resulting from such causes, the date of the delivery shall be extended for a period equal to the time lost by reason of the delay.

PATENTS

The Company will indemnify the Purchaser against any claim that goods manufactured according to a Company design and sold under this order infringe any United States patent or patent right. The Purchaser will indemnify the Company against all claims of patent infringement with respect to goods manufactured wholly or partially to the Purchaser's designs or specifications. Save as provided otherwise in research and development contracts all proprietary rights in designs, tools, patterns, drawings, data and equipment not furnished by the Purchaser are reserved to the Company.

TAXES

The amount of any sales or excise taxes, and import or export tariffs, levied on the Company and applicable to the equipment sold hereunder, (a) if payable in U.S. dollars shall be added to the above price and shall be paid by the Purchaser in the same manner and with the same effect as if originally added thereto, (b) if payable in any other currency, shall be payable by the Purchaser when and as incurred. In either case the Purchaser may, in lieu of payment, furnish the Company with a tax exemption certificate acceptable to the taxing authorities.

TRAVEL

All fixed price quotations are exclusive of expenses incurred by the Company on trips made at the request of the customer for the purposes of consultation, field inspection or installation.

WARRANTY

The performance of optical systems manufactured by the Company in accordance with its designs is warranted under the conditions of operation specified in the technical quotation. The Company also warrants all mechanical parts of its equipment, except stock components not manufactured by the Company, against defects of material and workmanship, for ninety (90) days from date of delivery; optical components are warranted against defects of workmanship on the date that they are accepted by the Purchaser. Save for warranty of title, no other warranty shall be implied.

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[redacted]

PROPRIETARY RIGHTS IN BID INFORMATION

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[redacted] reserves all right, title, and interest in background inventions, improvements and discoveries disclosed herein which are conceived and reduced to practice prior to the award of a contract by the offeree.

The data furnished shall be deemed a confidential disclosure and may not be disclosed outside the offeree's facility or be duplicated, used or disclosed in whole or in part for any purpose other than to evaluate the proposal; provided, that if a contract is awarded to this company as a result of or in connection with the submission of such data, the offeree shall have the right to duplicate, use or disclose this data to the extent provided in the contract. This restriction does not limit the offeree's right to use information contained in such data if it is obtained from another source.

MEMORANDUM

TO: All Employees

STATINTL FROM: [redacted]

SUBJ: Travel Policy

The following policies shall be in effect for all travel by employees which is made at the request of the Company:

A. Transportation by commercial carrier:
Tickets will be purchased by the Company through Business Administration.

B. Transportation by private automobile:
1. Travel within a metropolitan [redacted] area, or such other city driving as may be within the scope of the employees duties, will be reimbursed at ten cents per mile. 25X1
2. Highway travel will be reimbursed at seven cents per mile.

C. Per diem allowance:
The Company will provide either a $20.00 per diem allowance or pay actual itemized expenses. Receipts should be supplied whenever possible.

A travel advance may be secured from Business Administration if the trip is expected to exceed one day's duration. Travel vouchers must be submitted in order to secure reimbursement.

Employees should retain a copy of the travel voucher for their own tax records.

Form: 7163B-2

[redacted]

BUSINESS HISTORY

STATINTL [redacted] was incorporated in November 1959, under the laws of the [redacted] Its founding technical personnel were all previously associated with the [redacted] which was subsequently acquired by [redacted] 25X1 STATINTL STATINTL

It is a small business principally devoted to the design and fabrication of precision optical components and systems for industry, education, space exploration and defense. It is internationally recognized as a leading manufacturer of complex optics and its research staff of physicists and engineers includes one of the most distinguished optical design departments in the United States, if not the world. The business of the Company is primarily on a custom basis. Proprietary items are fabricated on a short lead time. In the component field its primary manufactured items are lenses, mirrors, windows and prisms, and the mechanical mountings and structures with which they are associated. Optics are generated from glass, quartz, beryllium, germanium and other optical materials. In the systems field the Company designs and manufactures optical instrumentation such as collimators and theodosyns. Projects have ranged from the polishing of tiny prisms to the design and fabrication of an extraordinary 155° ultra wide angle camera lens for [redacted] from a relatively simple coronograph to a two ton massive reducer with a resolving power approximating 400 lines per millimeter. STATINTL

STATINTL Although [redacted] is uniquely involved in the design and prototype manufacture of complex optics, moderate production runs, the fabrication of large optics and systems development can be easily accommodated at the [redacted] laboratories. STATINTL

Precision optics requires a rare combination of arts and crafts. It is unfortunately true that optical designers and engineers are in short supply as the subject shows signs of disappearing from the undergraduate and graduate curriculum. Precision opticians are equally rare, as hand craftmanship is all but abandoned by American labor. At [redacted] we have assembled, trained and extended the high level of intelligence and consummate skill equal to the challenge of a fine art. STATINTL

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OPTICS

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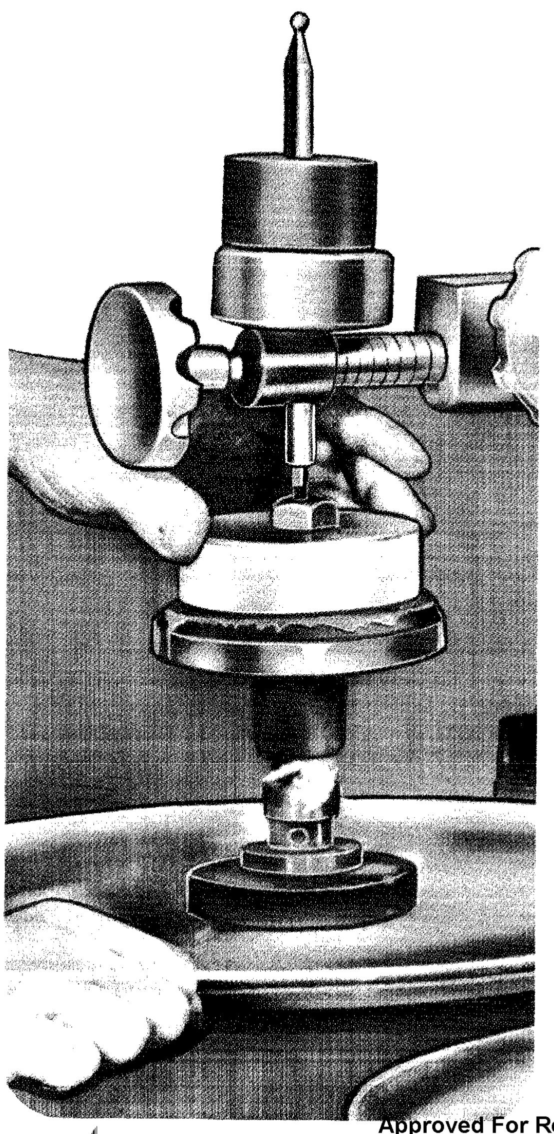
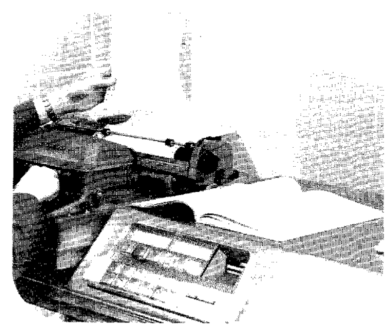
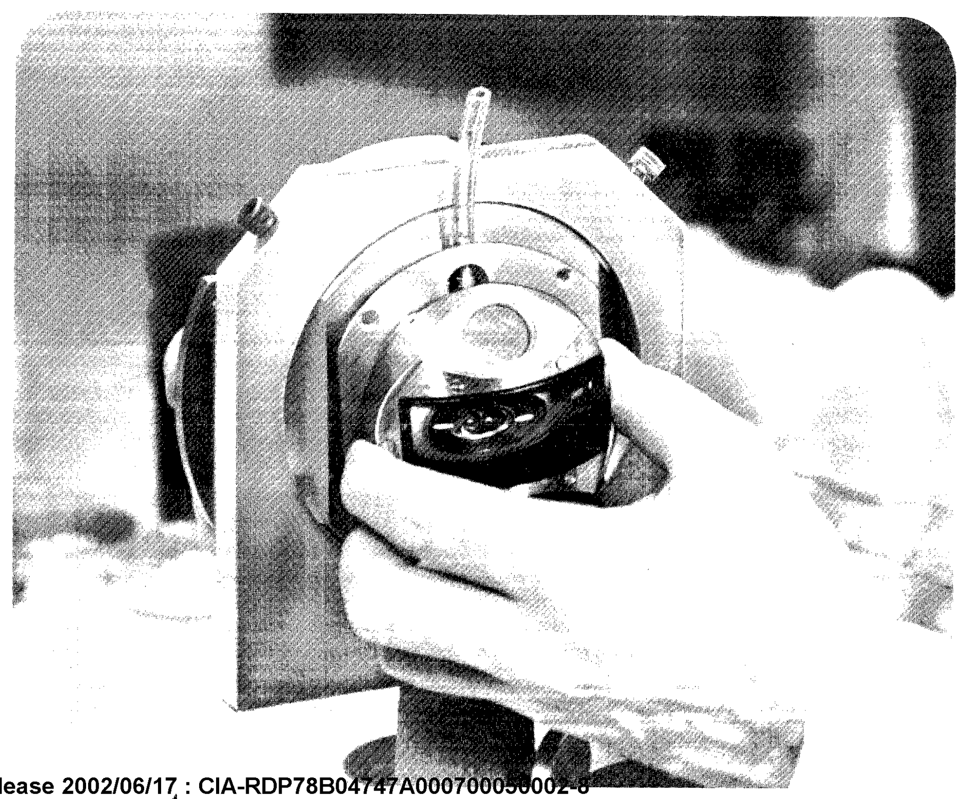




Hands at work



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